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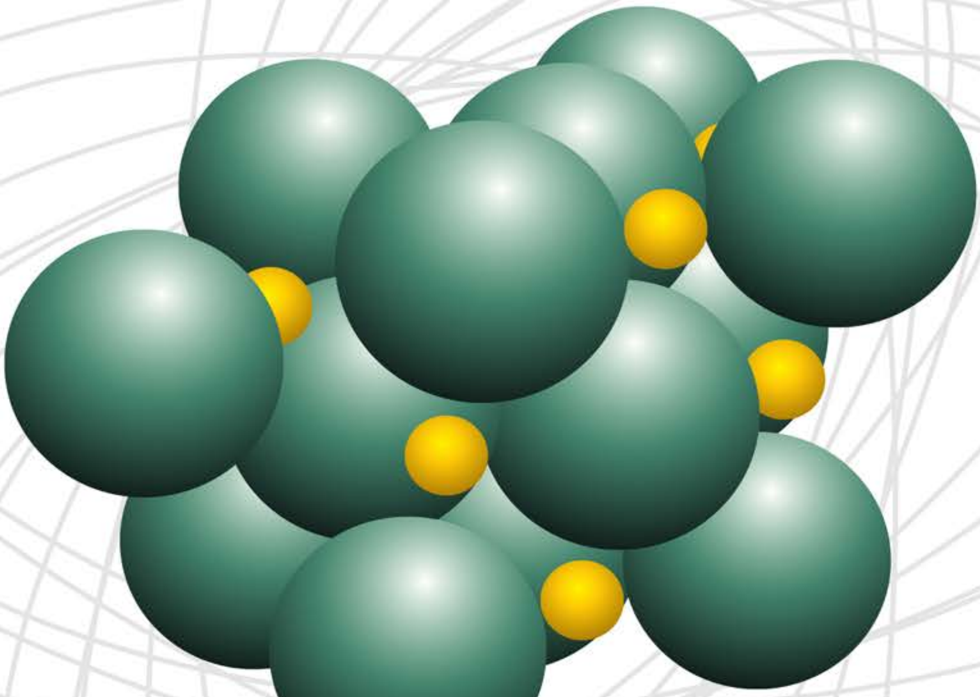

S. Mani Naidu

A text book of

ENGINEERING PHYSICS-I

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Brief Contents

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Preface

The contents of *Engineering Physics-I* have been designed to cater to the needs of B.Tech. students at the Jawaharlal Nehru Technological University Kakinada (JNTUK), Kakinada. The book follows a simple narrative style with emphasis on clarity. The concepts are treated rigorously to help students gain a deep-seated understanding of the key elements intrinsic to the subject. To this end, a list of important formulae, solved problems, multiple-choice questions and review questions have been included at the end of each chapter. These pedagogical elements would prepare the student-reader to face both internal tests and term-end examinations with ease.

Engineering Physics-I deals with the physics of substances that are of practical utility. This book focuses on the complete JNTUK syllabus that includes interference, diffraction, polarization, crystal structures, X-ray diffraction, lasers, fiber optics and non-destructive testing using ultrasonics.

I hope this book will be beneficial to both students and teachers of physics at various engineering colleges. Comments, feedback and suggestions for the improvement of this book are welcome. Any error that may have crept into the book inadvertently may kindly be brought to my notice or to that of the publisher.

Mani Naidu

Acknowledgements

I thank Dr M. Mohan Babu, cine artiste, former MP and chairman, Sree Vidyanikethan educational institutions, for providing a congenial atmosphere and encouraging me to write this book.

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I appreciate the contribution of the team at Pearson Education, especially that of Thomas Mathew Rajesh, M. E. Sethurajan, Sojan Jose and Jennifer Sargunar in bringing this work to fruition.

I am indebted to my guide, Dr M. Krishnaiah, Professor of physics, S. V. University, Tirupati, for his valuable suggestions to enhance the worth of this book. I am also grateful to all my students and colleagues at SVEC for evincing keen interest in the making of this book.

Last, but not the least, I am eternally thankful to the goddess Sree Sallapuramma for granting me the perseverance and commitment to complete this book.

Mani Naidu

Roadmap to the Syllabus

Engineering Physics-I
Jawaharlal Nehru Technological University Kakinada
Kakinada

UNIT I INTERFERENCE

Superposition of waves – Young's double slit experiment – Coherence – Interference in thin films by reflection – Newton's rings.

REFER **Chapter 1**

UNIT II DIFFRACTION

Fresnel and Fraunhofer diffractions – Fraunhofer diffraction at a single slit – Double slit – Diffraction grating – Grating spectrum – Resolving power of a grating – Rayleigh's criterion for resolving power.

REFER **Chapter 2**

UNIT III POLARIZATION

Types of Polarization – Double refraction – Nicol prism – Quarter wave plate and half wave plate.

REFER **Chapter 3**

UNIT IV CRYSTAL STRUCTURE

Introduction – Space lattice – Basis – Unit cell – Lattice parameters – Bravais lattices – Crystal systems – Structure and packing fractions of simple cubic, Body centered cubic, Face centered cubic crystals.

REFER **Chapter 4**

UNIT V X-RAY DIFFRACTION

Directions and planes in crystals – Miller indices – Separation between successive [$h\ k\ l$] planes – Diffraction of X-rays by crystal planes – Bragg's law – Laue method – Powder method.

REFER **Chapter 5**

UNIT V LASERS

Introduction – Characteristics of lasers – Spontaneous and Stimulated emission of radiation – Einstein's coefficients – Population inversion – Ruby laser – Helium – Neon laser – Semiconductor laser – Applications of lasers in industry, scientific and medical fields.

☞ REFER **Chapter 6**

UNIT VI FIBER OPTICS

Introduction – Principle of optical fiber – Acceptance angle and acceptance cone – Numerical aperture – Types of optical fibers and refractive index profiles – Attenuation in optical fibers – Application of optical fibers.

☞ REFER **Chapter 7**

UNIT VII NON-DESTRUCTIVE TESTING USING ULTRASONICS

Ultrasonic testing – Basic principle – Transducer – Couplant and inspection standards – Inspection methods – Pulse echo testing technique – Flaw detector – Different types of scans – Applications.

☞ REFER **Chapter 8**

CHAPTER 1

Interference

1.1 Introduction

When a number of waves pass through a point in a medium at the same time, they combine to produce a resultant wave having different amplitude and hence intensity than the individual waves at that point. This process is known as the interference. The resultant intensity at a point is due to the combined influence of all the waves that pass through the point. Interference produces modification in the distribution of intensity of light. Interference have been observed with light waves, sound waves, water waves, etc. Interference can be explained using the principle of superposition of waves.

1.2 Superposition of waves

The principle of superposition of waves states that the resultant displacement at a point in a medium is the algebraic sum of the displacements due to individual waves passing through that point simultaneously. Superposition takes place when two or more waves pass through a point in a medium.

To understand superposition, we consider two waves passing through a point in a medium. Let y_1 be the displacement of a particle at that point due to the first wave in the absence of second wave and y_2 be the displacement at that point due to the second wave in the absence of the first wave.

The resultant displacement (R) at that point when both the waves act simultaneously is

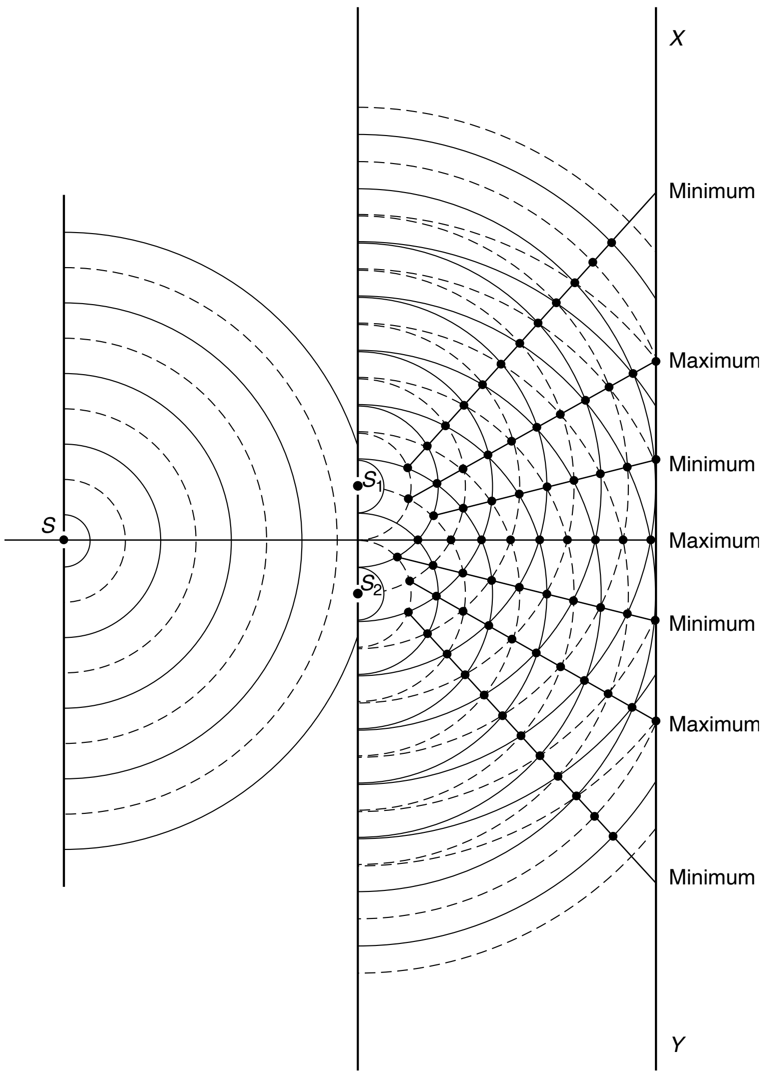
$$R = y_1 \pm y_2$$

The '+' sign is used when both displacements are in the same direction and the '−' sign is used when the two displacements are in opposite directions.

1.3 Young's double slit experiment

In 1801, Thomas Young first demonstrated interference of light. His experimental set-up is shown in Fig. 1.1. As shown in Fig. 1.1, he passed sun light through a narrow pin hole 'S' on an opaque surface and then through two closely spaced pin holes S_1 and S_2 in another opaque surface. A screen 'XY' was arranged in front of the pin holes S_1 and S_2. He observed a few coloured bright and dark interference bands on the screen. To obtain a large number of clear bright and dark interference bands, the sun light in the experiment was replaced by monochromatic light and the pin holes were replaced by narrow slits.

Figure 1.1 Young's double slit experimental set-up



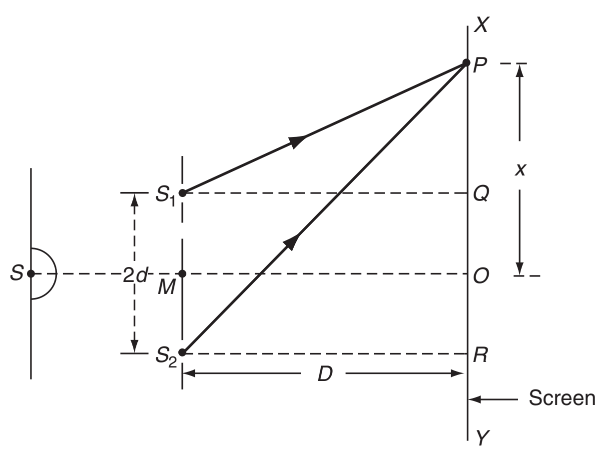
Explanation of interference

The interference has been explained based on the wave theory of light. As light passes through the pin hole 'S', spherical wave fronts spread out as shown in Fig. 1.1. The solid curves represent crests and dotted curves represent troughs on the waves. According to Huygen's wave theory, every point on the surface of the wave front acts as a source of secondary wavelets. As the wave front spreads to pin holes S_1 and S_2, the wave front at S_1 and S_2 act as secondary sources. The radii of these secondary wave fronts increase as they go away from S_1 and S_2 and superimpose to produce interference. At a point if a crest of one wave falls on the crest of another wave, [or a trough of one wave combines with a trough of another wave] then the amplitude of the resultant wave is the sum of amplitudes of the two crests (or troughs). This causes constructive interference at that point and a bright interference fringe is observed at that point. On the other hand if a crest of one wave falls on the trough of another wave then destructive interference takes place at that point and the resultant amplitude at that point is minimum, so a dark interference fringe is observed at that point. Since the intensity (I) of light is proportional to square of amplitude ($I \alpha R^2$) the intensity is large at the places where constructive interference takes place and the intensity is minimum at the places where destructive interference takes place. On the screen a large number of alternatively bright and dark interference bands (or fringes) of equal width are observed.

Analytical treatment of interference

As shown if Fig. 1.2, a source of monochromatic light of wave length 'λ' is passed through a pin hole 'S'. Spherical wave fronts were diverge and pass through the pin holes S_1 and S_2. The wave fronts in the pin holes S_1 and S_2 act as a source of secondary wavelets. The secondary wave fronts from S_1 and S_2 combine to produce an interference pattern on the screen 'XY'. Now we will determine the intensity of interference fringe at a point P on the screen. The screen is at a distance of D from S_1 and S_2.

Figure 1.2 **Schematic representation of interference due to two slits**



Let the amplitudes of the waves from S_1 and S_2 be a_1 and a_2. Let the phase difference between the waves at P be ϕ. Suppose that Y_1 and Y_2 are the displacements of these waves at P

Then $Y_1 = a_1 \sin \omega t$ ___________ (1.1)

and

$Y_2 = a_2 \sin (\omega t + \phi)$ ___________ (1.2)

These two waves combine to produce a resultant wave having displacement Y, given as

$$Y = Y_1 + Y_2 = a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

$$= \sin \omega t \, [a_1 + a_2 \cos \phi] + \cos \omega t \, (a_2 \sin \phi) \quad \text{\_\_\_\_\_\_\_\_\_\_\_ (1.3)}$$

Let $R \cos \theta = a_1 + a_2 \cos \phi$ ___________ (1.4)

and

$R \sin \theta = a_2 \sin \phi$ ___________ (1.5)

Substituting Equations (1.4) and (1.5) in (1.3), we get

$$Y = \sin \omega t \cdot R \cos \theta + \cos \omega t \cdot R \sin \theta$$

$$= R \sin (\omega t + \theta) \quad \text{\_\_\_\_\_\_\_\_\_\_\_ (1.6)}$$

Equation 1.6 represents the resultant wave having amplitude R. This can be obtained by squaring and adding Equations (1.4) and (1.5)

$$R^2(\cos^2\theta + \sin^2\theta) = R^2 = a_1^2 + a_2^2 \cos^2 \phi + 2a_1a_2 \cos\phi + a_2^2 \sin^2 \phi$$

$$= a_1^2 + a_2^2(\cos^2 \phi + \sin^2 \phi) + 2a_1a_2 \cos\phi$$

$$= a_1^2 + a_2^2 + 2a_1a_2 \cos\phi \quad \text{\_\_\_\_\_\_\_\_\_\_\_ (1.7)}$$

Since intensity (I) of light is equal to square of amplitude, Equation (1.7) represents the intensity of light at 'P'.

Hence $I = R^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$ ___________ (1.8)

The phase difference between the two waves at 'P' can be represented in terms of path difference as

$$\phi = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} \times (S_2P - S_1P) \quad \text{\_\_\_\_\_\_\_\_\_\_\_ (1.9)}$$

Based on the phase or path difference we see the following special cases for intensity of interference fringes.

Case (1): *Condition for maximum intensity*: For maximum intensity
Cos $\phi = 1$ in Equation (1.8), then

$$\phi = 0, 2\pi, 4\pi, \ldots = 2n\pi \text{ where } n = 0, 1, 2, \ldots$$

or the path difference $S_2P - S_1P = n\lambda$
Then the intensity of light at 'P' is

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

This is larger than the sum of the individual intensities of waves i.e., $a_1^2 + a_2^2$. Suppose $a_1 = a_2 = a$ then $I_{max} = 4a^2$.

Case (2): *Condition for minimum intensity*: For minimum intensity
$\cos\phi = -1$ in Equation (1.8), then

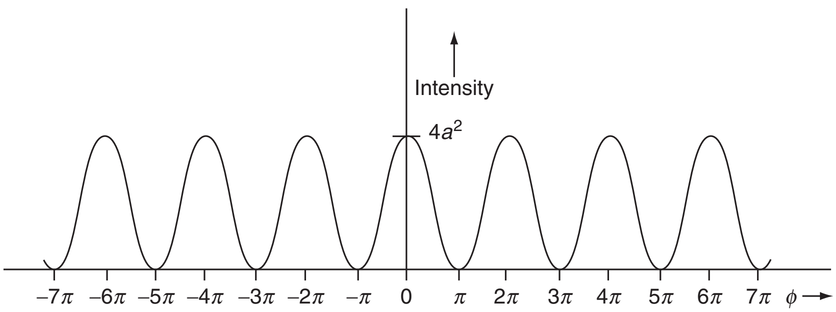
$$\phi = \pi, 3\pi, 5\pi, ... = (2n-1)\pi \text{ where } n = 1, 2, 3, ...$$

or the path difference $(S_2P - S_1P) = (2n-1)\lambda/2$

Now the intensity of light at 'P' is $I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$
Suppose $a_1 = a_2 = a$ then intensity of light is zero.

The above conditions shows that the intensity of light at bright points is $4a^2$ whereas at dark points intensity is zero. According to conservation of energy, the intensity of light is redistributed as shown in Fig. 1.3.

Figure 1.3 Distribution of intensity of light at different points due to interference



Theory of interference fringes

Let us now find the width of bright and dark interference fringes. As shown in Fig. 1.2, the point 'O' on the screen is equidistant from S_1 and S_2. Hence, the path difference between the waves reaching 'O' from S_1 and S_2 is zero. So, the intensity of light at 'O' is maximum. Let S_1 and S_2 be separated by a distance $2d$. Now we consider a point 'P' at a distance 'x' from 'O' on the screen. The condition for bright or dark fringe at 'P' depends on the path difference between S_1P and S_2P.
These paths can be obtained as follows:
From the right angle triangle S_1QP

$$(S_1P)^2 = (S_1Q)^2 + (QP)^2$$
$$= D^2 + (x-d)^2 \quad (\because QP = OP - OQ = x - d)$$

Similarly from the right angle triangle S_2RP

$$(S_2P)^2 = (S_2R)^2 + (RP)^2$$
$$= D^2 + (x+d)^2 \quad (\because RP = RO + OP = d + x)$$

$$\therefore (S_2P)^2 - (S_1P)^2 = (x+d)^2 - (x-d)^2 = 4xd$$

$$\text{or } (S_2P - S_1P)(S_2P + S_1P) = 4xd$$

$(S_2\,P - S_1\,P)\cdot 2D = 4xd$ [since $D \gg x$ or d so $S_2P \approx S_1P \approx D$]

or $(S_2P - S_1P) = \dfrac{4xd}{2D} = \dfrac{2xd}{D}$ ___________ (1.10)

Now we consider the following cases:

(1) Bright fringes: Suppose, the point P is bright then the path difference $S_2\,P - S_1\,P = n\lambda$ [where $n = 0, 1, 2, \ldots$]

Substituting the above value in Equation (1.10), we have

$$n\lambda = \frac{2xd}{D} \quad \text{or} \quad x = \frac{n\lambda D}{2d}$$

At 'O' a bright fringe is present and the next bright fringes are found when $n = 1, 2, 3, \ldots$
i.e., when

$$n = 1 \qquad x_1 = \frac{\lambda D}{2d}$$

$$n = 2 \qquad x_2 = \frac{2\lambda D}{2d}$$

$$n = 3 \qquad x_3 = \frac{3\lambda D}{2d}$$

$$\vdots \qquad \vdots$$

$$n = n \qquad x_n = \frac{n\lambda D}{2d}$$

The distance between any two consecutive bright fringes is

$$x_{n+1} - x_n = \frac{\lambda D}{2d} \qquad \text{___________ (1.11)}$$

(2) Dark fringes: Suppose the point 'P' is dark, then the path difference $S_2P - S_1P = (2n-1)\lambda/2$, where $n = 1, 2, 3, \ldots$.

Substituting the above value in Equation (1.10), we have

$$\frac{2xd}{D} = (2n-1)\frac{\lambda}{2} \qquad \text{or} \qquad x = \frac{(2n-1)}{4d}\lambda D$$

Dark fringes are found at the following distances from 'O'

$$\text{when } n = 1 \qquad x_1 = \frac{\lambda D}{4d}$$

$$n = 2 \qquad x_2 = \frac{3\lambda D}{4d}$$

$$n = 3 \qquad x_3 = \frac{5\lambda D}{4d}$$

$$\vdots \qquad \vdots$$

$$n = n \qquad x_n = \frac{(2n-1)\lambda D}{4d}$$

This distance between any two consecutive dark fringes is

$$x_n - x_{n-1} = \frac{2\lambda D}{4d} = \frac{\lambda D}{2d} \quad \text{\_\_\_\_\_\_\_\_\_\_} (1.12)$$

From Equations (1.11) and (1.12) we know that the separation between any two consecutive bright or dark fringes is the same and is known as fringe width 'β'.

$$\therefore \beta = \frac{\lambda D}{2d}$$

1.4 Coherence

To produce interference, coherent sources of light are required. The concept of coherence is explained below. When two or more electromagnetic waves are said to be coherent, they have same frequency and are in phase or maintain a constant phase difference between them. In general, the phase can vary from point to point or can change from time to time. So we have two different kinds of coherences, namely (i) temporal coherence and (ii) spatial coherence.

(i) Temporal coherence: This refers to the correlation between the field of a wave at a point at some time and the field at the same point at a later time. For example, at a point (x, y, z), let the fields at times t_1 and t_2 be $E(x, y, z, t_1)$ and $E(x, y, z, t_2)$. If the phase difference between the two fields is constant during the period normally covered by observations, then the wave is said to be temporally coherent. On the other hand, if the phase difference changes many times and also irregularly during the short period of time, then it is said to be noncoherent.

(ii) Spatial coherence: If a constant phase difference exists at different points in space between the waves over a time t, then they are said to be spatially coherent. Temporal coherence refers to a single beam of light, whereas spatial coherence refers to the relationship between two separate beams of light.

1.5 Interference in thin films by reflection

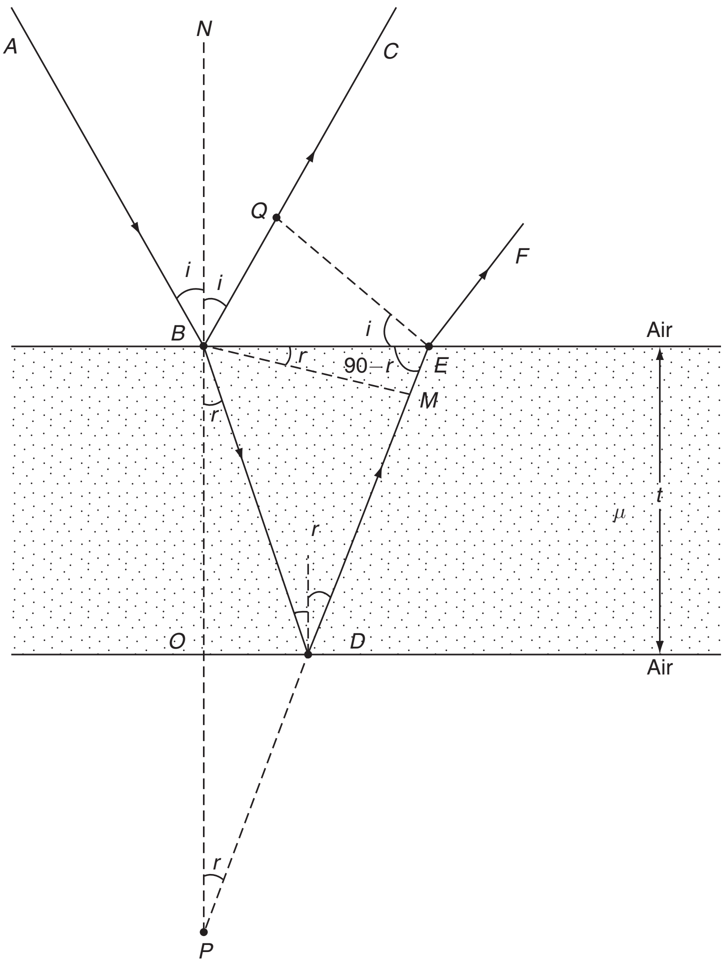
When white light (sun light) falls on thin films such as soap bubbles or air films then beautiful colours will appear due to interference. Interference of light in thin films can be explained by allowing monochromatic light of wave length 'λ' to fall on a thin film of thickness 't' as shown in Fig. 1.4. Let the refractive index of film material be 'μ'

AB is a monochromatic light beam incident at '*B*' on a uniform thick transparent film. A part of the beam get reflected along *BC* and the remaining light is refracted along *BD*. A part of the refracted ray gets reflected at '*D*' along *DE*. Again, this ray is refracted at '*E*' along *EF* and enters into air. The ray *BC* and *EF* are parallel and they superimpose to produce interference. The intensity of interference fringe depends on the path difference between the rays *BC* and *EF*. The path difference 'δ' is given below.

$$\begin{aligned} \delta &= \text{Path } (BD + DE) \text{ in film} - \text{path } BQ \text{ in air} \\ &= (BD + DE)\,\mu - (BQ)\,\mu_0 \\ &= (BD + DE)\mu - BQ \quad \text{\_\_\_\_\_\_\_\_\_\_} (1.13) \quad (\because \mu_0 = 1 \text{ for air}) \end{aligned}$$

The path difference can be simplified using Fig. 1.4.

Figure 1.4 **Interference of light due to thin film of thickness '*t*'**



A normal is drawn at B on the surface of the film, and extended backwards as NOP. Similarly the ray DE is extended backwards as EDP. The lines EP and BP will meet at P. A normal is drawn at E to BC and at B to DE.

From the figure,

$\angle ABN = i$ and $\angle NBC = i$. So $\angle QBE = 90° - i$. Hence, $\angle BEQ = i$ [because angle at Q in triangle BQE is 90°]

$\angle PBD = r$ and $\angle BDE = 2r$; $\angle DBE = 90° - r$

In triangle DBE, $\angle BED = 90° - r$. Hence in triangle MBE, $\angle MBE = r$

Also in triangle PBE, $\angle BPE = r$

In triangle BPD, the angles at B and P are equal, so $OB = OP$. The refractive index 'μ' in Equation (1.13) is given as $\mu = \dfrac{\sin i}{\sin r}$, In triangle BQE, $\sin i = \dfrac{BQ}{BE}$ and in triangle BME, $\sin r = \dfrac{ME}{BE}$

Hence $\mu = \dfrac{BQ/BE}{ME/BE} = \dfrac{BQ}{ME}$

or $BQ = \mu ME$ ____________ (1.14)

Substituting Equation (1.14) in (1.13), we have

$$\delta = (BD + DE)\mu - \mu ME$$
$$= (BD + DE - ME)\mu$$
$$= (PE - ME)\mu \quad [\text{since } BD = PD]$$
$$= (PM)\mu = \mu PB \cos r = [\text{since } \cos r = PM / PB]$$
$$= \mu.2t.\cos r = 2\mu t \cos r \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (1.15)}$$

The incident ray AB is reflected on the surface of a denser medium. Hence, it experiences an additional phase change π or path difference $\lambda/2$.

Hence the effective path difference between the rays BC and EF is

$$\delta = 2\mu t \cos r - \frac{\lambda}{2} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (1.16)}$$

Now, let us examine the condition for bright and dark interference fringes.

(i) If the path difference is equal to $n\lambda$

when $n = 1, 2, 3, \ldots$ then constructive interference occurs and the film appears bright

i.e. $2\mu t \cos r - \dfrac{\lambda}{2} = n\lambda$

or $2\mu t \cos r = n\lambda + \dfrac{\lambda}{2} = (2n+1)\dfrac{\lambda}{2}$ ____________ (1.17)

(ii) If the path difference is equal to $(2n - 1)\lambda/2$, then destructive interference occurs and the film appear dark

i.e. $2\mu t \cos r - \dfrac{\lambda}{2} = (2n-1)\dfrac{\lambda}{2}$

$2\mu t \cos r = (2n-1)\dfrac{\lambda}{2} + \dfrac{\lambda}{2} = n\lambda$ ____________ (1.18)

Equations (1.17) and (1.18) give the condition for bright and dark interference fringes in case of interference in thin films by reflection.

1.6 Newton's rings

As shown in Fig. 1.5, a plano convex lens (L) having long focal length, f ($\approx$100 cm) is placed with its convex surface on a plane glass plate (G). A gradually increasing thickness of air film will be formed between plane glass plate and convex surface of plano convex lens. The thickness of air film will be zero at the point of contact and symmetrically increases as we go radially from the point of contact. A monochromatic light of wave length 'λ' is allowed to fall normally on the lens with the help of a glass plate 'P' kept at 45° to the incident monochromatic

beam. A part of the incident light rays are reflected up at the convex surface of the lens and the remaining light is transmitted through the air film. Again a part of this transmitted light is reflected at on the top surface of the glass plate (G). Both the reflected rays combine to produce an interference pattern in the form of alternate bright and dark concentric circular rings, known as Newton's rings, because Newton first demonstrated and showed these rings. The rings are circular because the air film has circular symmetry. These rings can be seen through the travelling microscope M.

Figure 1.5a Experimental arrangement for Newton's rings

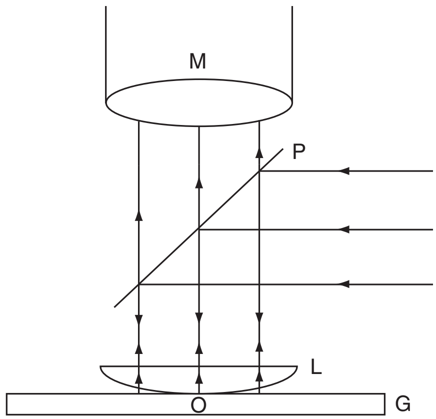
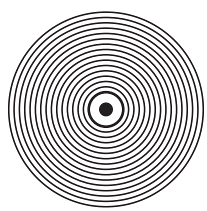


Figure 1.5b Newton's rings as seen through microscope



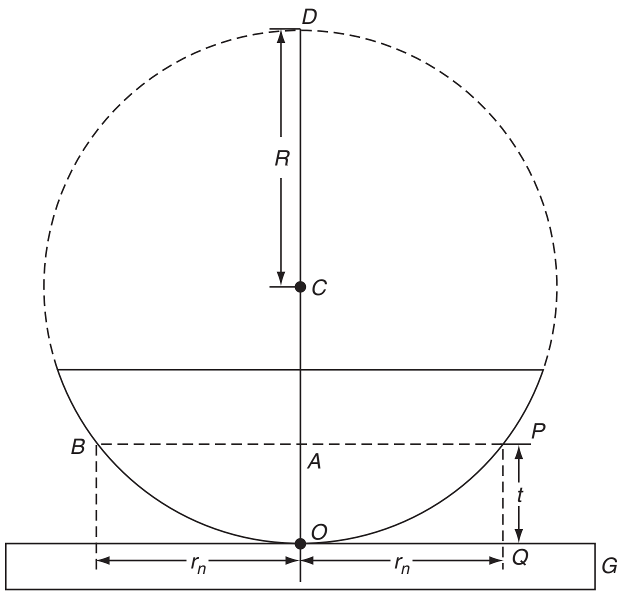
Theory: Newton's rings by reflected light

Let us discuss the interference condition for bright and dark fringes and also the spacing between Newton's rings. To obtain the relation between the radius of Newton's ring and the radius of curvature of the lens, consider Fig. 1.6.

Let the lens be in contact with glass plate at O and let the radius of curvature of the lens be R. Let a vertical light ray be partially reflected and partially transmitted at 'P'. The transmitted light is again reflected at Q on glass plate G. Let the thickness of air film at P be PQ $(= t)$ and the radius of Newton's ring at Q be r_n. The ray reflected at Q, suffers an additional phase change of π or path difference $\lambda/2$. The total path difference between the two reflected rays at 'P' and 'Q' is

$$\delta = 2t + \frac{\lambda}{2} \qquad \text{(1.19)}$$

Figure 1.6 Calculation of film thickness from radius of curvature of lens



The lens is a part of sphere of radius 'R' with centre 'C'. From the property of circle

$$BA \times AP = OA \times AD$$

Substituting the values

$$r_n \times r_n = t \times (2R - t) = 2Rt - t^2 \approx 2Rt$$

$$\therefore r_n^2 = 2Rt \text{ or } t = \frac{r_n^2}{2R} \quad \text{\_\_\_\_\_\_\_\_\_\_ (1.20)}$$

Substituting Equation (1.20) in (1.19) we have

$$\delta = 2\frac{r_n^2}{2R} + \frac{\lambda}{2} = \frac{r_n^2}{R} + \frac{\lambda}{2} \quad \text{\_\_\_\_\_\_\_\_\_\_ (1.21)}$$

If the path difference is $n\lambda$ then constructive interference takes place and a bright n^{th} Newton's ring will be formed.

i.e. $\delta = \dfrac{r_n^2}{R} + \dfrac{\lambda}{2} = n\lambda$ where $n = 1, 2, 3, \ldots$

or $\dfrac{r_n^2}{R} = (2n-1)\dfrac{\lambda}{2}$

or $r_n^2 = (2n-1)\dfrac{\lambda R}{2}$ __________ (1.22)

Expressing radius of the ring in terms of diameter 'D_n' of ring $(r_n = D_n/2)$ we have

$$r_n^2 = \left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2}$$

or $D_n^2 = \dfrac{4(2n-1)\lambda R}{2}$

or $D_n = \sqrt{2(2n-1)\lambda R}$ ____________ (1.23) [for bright ring]

Similarly for n^{th} dark ring, the path difference is equal to $(2n-1)\lambda/2$.

$\therefore$ From Equation (1.21)

$$\delta = \frac{r_n^2}{R} + \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

or $\dfrac{r_n^2}{R} = (2n-1)\dfrac{\lambda}{2} - \dfrac{\lambda}{2} = (n-1)\lambda = n\lambda$ [since $n-1$ and n are integers]

$\therefore r_n^2 = n\lambda R$ ____________ (1.24)

Expressing radius of ring in terms of diameter of ring $[r_n = D_n/2]$ we have

$$r_n^2 = \left(\frac{D_n}{2}\right)^2 = n\lambda R$$

$$D_n^2 = 4n\lambda R$$

or $D_n = \sqrt{4n\lambda R}$ ____________ (1.25) [for dark ring]

At the point of contact of lens and glass plate [at 'O'], the path difference is zero and phase change π takes place due to reflection on glass plate, hence dark spot will be formed at the centre of ring system.

From Equation (1.25) we know the diameter of rings is proportional to square root of the order of rings [i.e. $\sqrt{n}$]. Hence the spacing between consecutive rings goes on decreasing with an increase of order of rings.

The theory of Newton's rings can be used to determine the wave length of monochromatic light and the refractive index of a given liquid.

Application: Determination of wave length of monochromatic light

Using the experimental set up shown in Fig. 1.5(a), the diameters of various dark rings are measured using travelling microscope. To measure the diameter of rings, first the microscope is adjusted so that the centre of cross wires coincides with the centre of the ring system. The centre of the cross wires is moved to one direction (say left) so that the vertical cross wire is tangential to 21$^{\text{st}}$ dark ring (say). Now the reading is noted from the horizontal scale of travelling microscope. The microscope is moved towards right such that the vertical cross wire is tangential to 18$^{\text{th}}$ ring, then the reading is noted. By moving the microscope in the same direction, readings are noted for every 3 rings. Similar readings are noted for these rings on the right side also. The difference of readings on the left side and right side gives the diameter of the various rings.

Let the diameter of n^{th} and $(n+m)^{\text{th}}$ dark rings be D_n and D_{n+m}. Then

$$D_n^2 = 4n\lambda R \quad \text{and}$$

$$D_{n+m}^2 = 4(n+m)\lambda R$$

$$\therefore D_{n+m}^2 - D_n^2 = 4m\lambda R$$

or $\lambda = \dfrac{D_{n+m}^2 - D_n^2}{4mR}$ ____________ (1.26)

Substituting the average value of $\left(D_{n+m}^2 - D_n^2\right)/m$ in the above equation, 'λ' can be determined. R is the radius of curvature of the lens.

Application: Determination of refractive index of a given liquid

The experimental set up shown in Fig. 1.5(a) is used to find the refractive index of a given liquid. To find the refractive index of a liquid, the plane glass plate and plano convex lens set up is placed in a small metal container. The diameter of n^{th} and $(n + m)^{th}$ dark rings are determined, when there is air between plano convex lens and plane glass plate.
Then we have,

$$D_{n+m}^2 - D_n^2 = 4m\lambda R \qquad (1.27)$$

Now, the given liquid whose refractive index (μ) is to be measured is introduced into the space between plano convex lens and plane glass plate without disturbing the experimental set up. The optical path between plano convex lens and plane glass plate is not 't' but 'μt', hence the diameters of Newton's rings are changed. Now the diameters of n^{th} and $(n + m)^{th}$ dark rings are measured.

$$\text{Then } D'^2_{n+m} - D'^2_n = \frac{4m\lambda R}{\mu} \qquad (1.28)$$

where D'_n and D'_{n+m} represent the diameters of n^{th} and $(n + m)^{th}$ dark rings with liquid between plano convex lens and plane glass plate.

From Equations (1.27) and (1.28)
We have

$$\mu = \frac{D_{n+m}^2 - D_n^2}{D'^2_{n+m} - D'^2_n} \qquad (1.29)$$

Using Equation (1.29), the refractive index of a given liquid can be determined.

Formulae

1. $Y = Y_1 + Y_2 = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$
2. $I = R^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$
3. $I_{\max} = (a_1 + a_2)^2 = 4a^2$ if $a_1 = a_2 = a$
4. $I_{\min} = (a_1 - a_2)^2 = 0$ if $a_1 = a_2 = a$
5. $x_n = \dfrac{n\lambda D}{2d}$ for n^{th} bright fringe
6. $x_n = \dfrac{(2n-1)\lambda D}{4d}$ for n^{th} dark fring
7. Fringe width $(\beta) = \dfrac{\lambda D}{2d}$
8. $\delta = 2\mu t \cos r - \dfrac{\lambda}{2} = n\lambda$ for bright interference fringes
9. $2\mu t \cos r = n\lambda$ for dark interference fringes
10. $D_n = \sqrt{2(2n-1)\lambda R}$ for diameter of n^{th} bright ring
11. $D_n = \sqrt{4n\lambda R}$ for n^{th} dark ring
12. $\lambda = \dfrac{D_{n+m}^2 - D_n^2}{4mR}$
13. $\mu = \dfrac{D_{n+m}^2 - D_n^2}{D'^2_{n+m} - D'^2_n}$

Solved Problems

1. *Two coherent sources of intensity 10* W/m^2 *and 25* W/m^2 *interfere to from fringes. Find the ratio of maximum intensity to minimum intensity.*

[May 2004, Set No. 1; May 2003, Set No. 2]

Sol: We know the intensity (I) = a^2 [square of amplitude]

on this basis $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{10}{25}$

or $\frac{a_1}{a_2} = \frac{3.1623}{5}$

i.e. $a_1 = \left[\frac{3.1623}{5}\right] a_2 = 0.6324 a_2$

Ratio of I_{max} to I_{min} is

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{\left[0.6324\, a_2 + a_2\right]^2}{\left[06324\, a_2 - a_2\right]^2}$$

$$= \frac{\left(1.6324\, a_2\right)^2}{\left[-0.3675 a_2\right]^2} = \frac{2.6647}{0.1351} = 19.724$$

2. *In a double slit experiment a light of* $\lambda = 5460$ Å *is exposed to slits which are 0.1* mm *apart. The screen is placed 2* m *away from the slits. What is the angular position of the 10*th *maximum and 1*st *minimum*?

[June 2005, Set No. 3]

Sol: The given data are

Wave length of light (λ) = 5460 Å = 5460×10^{-10} m

Separation between slits ($2d$) = 0.1 mm = 1×10^{-4} m

Distance of screen (D) = 2 m

Angular position of 10th maximum ($\phi_{max\ 10}$) = ?

Angular position of 1st minimum ($\phi_{min\ 1}$) = ?

Condition for maximum intensity is $x_n = \frac{n\lambda D}{2d}$

Distance of 10th maximum, $X_{max\ 10} = \frac{10 \times 5460 \times 10^{-10} \times 2}{1 \times 10^{-4}}$

$= 0.1092$ m

10th maximum angular position, $\tan \phi_{max\ 10} = \frac{x_{max\ 10}}{D}$

$\therefore \tan \theta_{max\,10}\ \frac{0.1092}{2} = 0.0546$ radians

$= 3°7'37''$

Condition for minimum intensity is $x_n \dfrac{(2n-1)\lambda D}{4d}$

Distance of 1st minimum, $x_{\min 1} = \dfrac{\lambda D}{4d} = \dfrac{\lambda D}{2 \times 2d} = \dfrac{5460 \times 10^{-10} \times 2}{2 \times 10^{-4}}$

$$= 5460 \times 10^{-6} \text{ m}$$

Angular position of 1st minimum is $(\tan \phi_{\min 1}) = \dfrac{x_{\min 1}}{D}$

$$= \frac{5460 \times 10^{-6}}{2} = 0.00273 \,\text{radians} = 0.156^\circ = 0^\circ 9' 23''$$

3. *A soap film of refractive index 1.33 and thickness 5000 Å is exposed to white light. What wave lengths in the visible region are reflected?*

Sol: The given date are

Refractive index of soap (μ) = 1.33

Thickness of soap film (t) = 5000 Å = 5000×10^{-10} m

What wave lengths in the visible light are reflected?

The incident light reflected on both surfaces of film combine to produce interference. So, the condition for constructive interference is used for reflection.

i.e. $2\mu t \cos r - \dfrac{\lambda}{2} = n\lambda$

For maximum reflection $\cos r = 1$

then $2\mu t - \dfrac{\lambda}{2} = n\lambda$

or $\lambda = \dfrac{4\mu t}{2(n+1)}$

Substituting values,

$$\lambda = \frac{4 \times 1.33 \times 5000 \times 10^{-10}}{2n+1} = \frac{26600 \times 10^{-10}}{2n+1} \text{ m}$$

For $n = 0$ $\quad \lambda_1 = 26600 \times 10^{-10}$ m (IR region)

$n = 1$ $\quad \lambda_2 = \dfrac{26600 \times 10^{-10}}{3}$ m $= 8666.67 \times 10^{-10}$ m (IR region)

$n = 2$ $\quad \lambda_3 = \dfrac{26600 \times 10^{-10}}{5}$ m $= 5320 \times 10^{-10}$ m (Visible region)

$n = 3$ $\quad \lambda_4 = \dfrac{26600 \times 10^{-10}}{7}$ m $= 3800 \times 10^{-10}$ m (UV region)

From the above wave lengths, 5320 Å lies in the visible region. This wave length of white light is reflected maximum.

4. *In a Newton's rings experiment the diameter of the 15th ring was found to be 0.59* cm *and that of the 5th ring is 0.336* cm. *If the radius of curvature of the lens is 100* cm, *find the wave length of the light.*

[June 2005, Set No. 2]

Sol: The given data are

Diameter of Newton's 15th ring (D_{15}) = 0.59 cm = 0.59×10^{-2} m

Diameter of Newton's 5th ring (D_5) = 0.336 cm = 0.336×10^{-2} m

Radius of curvature of lens (R) = 100 cm = 1 m

Wave length of light (λ) = ?

$$\lambda = \frac{D^2_{n+m} - D^2_n}{4mR} = \frac{\left(0.59 \times 10^{-2}\right)^2 - \left(0.336 \times 10^{-2}\right)^2}{4 \times 10 \times 1}$$

$$= \frac{0.3481 \times 10^{-4} - 0.112896 \times 10^{-4}}{40} = \frac{0.235204 \times 10^{-4}}{40}$$

$$= 0.00588 \times 10^{-4}\,\text{m} = 5880 \times 10^{-10}\,\text{m} = 5880\,\text{Å}$$

5. *Newton's rings are observed in the reflected light of wave length 5900* Å. *The diameter of 10th dark ring is 0.5* cm. *Find the radius of curvature of the lens used.*

[June 2005, Set No. 4]

Sol: The given data are

Wave length of light (λ) = 5900 Å= 5900×10^{-10} m

Diameter of 10th Newton's dark ring (D_{10}) = 0.5 cm = 0.5×10^{-2} m

Radius of curvature of lens (R) = ?

Formula is $D^2_n = 4n\lambda R$

$$\text{or } R = \frac{D^2_n}{4n\lambda} = \frac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 5900 \times 10^{-10}}$$

$$= \frac{0.25 \times 10^{-4}}{236 \times 10^{-7}} = 1.059\ \text{m}$$

6. *Light waves of wave length 650* nm *and 500* nm *produce interference fringes on a screen at a distance of 1* m *from a double slit of separation 0.5* mm. *Find the least distance of a point from the central maximum where the bright fringe due to both sources coincide.*

[June 2004, Set No. 2; May 2003, Set No. 3]

Sol: Given data are

Wave length of first source (λ_1) = 650 nm

Wave length of second source (λ_2) = 500 nm

Distance of screen (D) = 1 m

Separation between slits ($2d$) = 0.5 mm = 0.5×10^{-3} m

Distance from central maximum where bright fringes due to both sources coincide (x) = ?

Let us consider the n^{th} bright fringe of the first source and the m^{th} bright fringe of the second source coincide at a distance of 'x' from central maximum.

Then,

$$x = \frac{n\lambda_1 D}{2d} = \frac{m\lambda_2 D}{2d}$$

or,

$$n\lambda_1 = m\lambda_2$$

or,

$$\frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{500}{650} = \frac{10}{13}$$

$\therefore$ 10th bright fringe due to the first source coincides with 13th bright fringe due to second source.

Also, $x = \dfrac{n\lambda_1 D}{2d} = \dfrac{10 \times 650 \times 10^{-9} \times 1}{0.5 \times 10^{-3}} = 0.013\,\text{m} = 13\,\text{mm}$

$\therefore$ The bright fringes of both sources will coincide at a distance of 13 mm from central maximum.

7. *Calculate the thickness of air film at the 10th dark ring in a Newton's rings system, viewed normally by a reflected light of wave length 500* nm. *The diameter of the 10th dark ring is 2* mm.

[June 2004, Set No. 4]

Sol: Given data are

Wave length of light (λ) = 500 nm = 500 × 10^{-9} m

Number of the dark ring viewed (n) = 10

Diameter of 10th dark ring (D_{10}) = 2 mm = 2 × 10^{-3} m

Radius of 10th dark ring (r_{10}) = $\dfrac{D_{10}}{2} = 1 \times 10^{-3}$ m

Thickness of air film (t) = ?

Condition for dark ring is

$$D_n = \sqrt{4n\lambda R}$$

or $D_n^2 = 4n\lambda R$

or $R = \dfrac{D_n^2}{4n\lambda} = \dfrac{(2 \times 10^{-3})^2}{4 \times 10 \times 500 \times 10^{-9}} = \dfrac{4 \times 10^{-6}}{40 \times 500 \times 10^{-9}} = 0.2$ m

Also $t = \dfrac{r_n^2}{2R} = \dfrac{(10^{-3})^2}{2 \times 0.2} = 2.5 \times 10^{-6}\,\text{m} = 2.5\ \mu\text{m}$

8. *Two slits separated by a distance of 0.2* mm *are illuminated by a monochromatic light of wave length 550* nm. *Calculate the fringe width on a screen at a distance of 1* m *from the slits.*

[June 2004, Set No. 3]

Sol: The given data are

Separation between the slits ($2d$) = 0.2 mm = 0.2 × 10^{-3} m

Wave length of light (λ) = 550 nm = 550 × 10^{-9} m

Distance of the screen $(D) = 1$ m

Fringe width $(\beta) = ?$

$$\beta = \frac{\lambda D}{2d} = \frac{500\times10^{-9}\times1}{0.2\times10^{-3}} = \frac{500\times10^{-6}}{0.2} = 2750\times10^{-6}\,\text{m}$$
$$= 2.75\times10^{-3}\,\text{m} = 2.75\text{ mm}$$

9. *Light of wave length 500* nm *forms an interference pattern on a screen at a distance of 2* m *from the slit. If 100 fringes are formed within a distance of 5* cm *on the screen, find the distance between the slits.*

[May 2003, Set No. 4]

Sol: The given data are

Wave length of light $(\lambda) = 500$ nm $= 500 \times 10^{-9}$ m

Distance of screen $(D) = 2$ m

Fringe width $(\beta) = \dfrac{5}{100}\text{ cm} = \dfrac{50}{100}\text{ mm} = 0.5\,\text{mm}$

$= 0.5 \times 10^{-3}\,\text{m}$

Separation between slits $(2d) = ?$

We know $\beta = \dfrac{\lambda D}{2d}$

or $2d = \dfrac{\lambda D}{\beta} = \dfrac{500\times10^{-9}\times2}{0.5\times10^{-3}} = 2\times10^{-3}\,\text{m} = 2\text{ mm}$

10. *Two coherent sources whose intensity ratio is 36:1 produce interference fringes. Deduce the ratio of maximum intensity to minimum intensity.*

[May 2008, Set No. 2]

Sol: Intensities ratio of coherent sources $= a_1^2 : a_2^2 = 36:1$

$\therefore a_1 : a_2 = 6:1$

Minimum intensity of the interference fringe $= (a_1 - a_2)^2$

$= (6 - 1)^2 = 25$

Maximum intensity of the interference fringe $= (a_1 + a_2)^2$

$= (6 + 1)^2 = 49$

The ratio of maximum intensity to minimum intensity $= 49:25 \approx 2:1$

11. *In a Newton's ring experiment, the diameter of the 5*th *ring is 0.30* cm *and diameter of the 15*th *ring is 0.62* cm. *Find the diameter of the 25*th *ring.*

[June 2009, Set No. 3]

Sol: Diameter of Newton's 5th ring = 0.30 cm

Diameter of Newton's 15th ring = 0.62 cm

and Diameter of Newton's 25th ring = ?

From Newton's rings experiment we know

$$\lambda = \frac{D^2_{n+m} - D^2_n}{4m\mathrm{R}} \text{\_\_\_\_\_\_\_\_}(14.26)$$

or $D^2_{n+m} - D^2_n = 4\lambda mR$

For 5th and 15th rings

$D^2_{15} - D^2_5 = 4\lambda \times 10 \times R$ ________ (1) $(m = 10)$

For 15th and 25th rings

$D^2_{25} - D^2_{15} = 4\lambda \times 10 \times R$ ________ (2) $(m = 10)$

Equation (2) = Equation (1)

$D^2_{25} - D^2_{15} = D^2_{15} - D^2_5$

or $D^2_{25} = 2D^2_{15} - D^2_5$

Substituting the values,

$$D^2_{25} = 2\times 0.62\times 0.62 - 0.3\times 0.3 = 0.6788 \text{ cm}^2$$

$\therefore \mathrm{D}_{25} = 0.8239$ cm

Multiple-choice Questions

1. The process of waves appearing with different intensity at a point when a number of waves pass through a point in a medium is known as __________ .

   (a) interference (b) diffraction (c) polarization (d) all

2. Interference of waves have been observed with __________ .

   (a) light waves (b) sound waves (c) water waves (d) all

3. Intensity of light is proportional to __________ of amplitude.

   (a) double (b) square (c) square root (d) cube root

4. If a_1 and a_2 are the amplitudes of two sources in Young's double slit experiment, then the maximum intensity of interference fringe is __________ .

   (a) $(a_1 + a_2)$ (b) $2(a_1 + a_2)$ (c) $(a_1 + a_2)^2$ (d) $(a_1 - a_2)^2$

5. If a_1 and a_2 are the amplitudes of two sources in Young's double slit experiment then the minimum intensity of interference fringe is __________ .

   (a) $(a_1 + a_2)^2$ (b) $(a_1 - a_2)^2$ (c) $(a_1 + a_2)^3$ (d) $2(a_1 + a_2)$

6. In Young's double slit experiment, two slits are separated by a distance of '$2d$' and a screen is placed at a distance of 'D' from slits. If a monochromatic light source of wave length 'λ' is incident on the slits, then the fringe width 'β' is equal to __________ .

   (a) $\dfrac{\lambda D}{2d}$ (b) $\dfrac{\lambda D}{d}$ (c) $\dfrac{2d}{\lambda D}$ (d) $\dfrac{\lambda d}{2D}$

7. In Young's double slit experiment, if the separation between slits is reduced to half and the distance of the screen from the slits is doubled, then the fringe width is ___________ .

(a) unchanged (b) halved (c) doubled (d) quadrupled

8. If the Young's double slit experiment is moved from air medium to water medium then the fringe pattern is ____ .

(a) enlarged (b) shrink (c) unchanged (d) none

9. In Young's double slit experiment, two waves having intensities in the ratio of 9 : 1 produce interference. The ratio of maximum to minimum intensity is equal to ___________ .

(a) 4 : 1 (b) 2 : 1 (c) 9 : 1 (d) 3 : 1

10. In Young's double slit experiment, the slits are at a separation of 0.6 mm and the screen is at a distance of 1 m from the slits. If the slits are illuminated by a wave length of 600 nm then the distance of the 5th dark fringe is _______ .

(a) 4.5 mm (b) 5 mm (c) 45 mm (d) 50 mm

11. In Young's double slit experiment, a fringe width of 0.6 mm is obtained when 600 nm wave length is used. If the wave length is changed to 400 nm, then the fringe width is ___________ .

(a) 0.6 mm (b) 0.4 mm (c) 0.24 mm (d) 0.48 mm

12. If two waves have the same frequency and they are in phase or maintain constant phase relationship, then they are said to be ___________ .

(a) coherent (b) non coherent (c) both *a* and *b* (d) none

13. If a ray is reflected on the surface of a denser medium, then it experiences an additional phase change of _______ .

(a) $\frac{\pi}{2}$ (b) 2π (c) π (d) $\frac{3\pi}{4}$

14. When white light is incident on a thin film of oil or soap bubble, then multiple colours appear due to ___________ of light.

(a) interference (b) diffraction (c) polarization (d) none

15. Newton's rings are ___________ .

(a) circular` (b) parabolic (c) hyperbolic (d) none

16. Newton's rings are circular because the air space between plane glass plate and plano convex lens is ___________ .

(a) circularly non-symmetric (b) circularly symmetric
(c) parabolically symmetric (d) parabolically non-symmetric

17. If 'R' is the radius of curvature of the plano convex lens and 'λ' is the wave length of monochromatic light incident normally in a Newton's ring experiment, the expression for diameter of n^{th} dark ring is ___________ .

(a) $4n\lambda R$ (b) $2n\lambda R$ (c) $\sqrt{2n\lambda R}$ (d) $\sqrt{4n\lambda R}$

18. The dark spot at the centre of Newton's rings is formed because ___________ .

(a) the convex surface of the lens is in contact with the plane glass plate.
(b) at the point of contact of the plano convex lens with the plane glass plate, the path difference is zero.
(c) at the point of contact of the convex surface with the plane glass plate, a phase change of 'π' takes place
(d) all

19. In Newton's rings, the spacing between consecutive rings ___________ with increase of order of rings.

(a) increases (b) remain constant (c) decreases (d) none

20. If D_n and D_{n+m} are the diameters of n^{th} and $(n + m)^{th}$ Newton's dark rings and R is the radius of curvature of plano convex lens. The expression for wave length of light is __________ .

(a) $\dfrac{D^2_{n+m} - D^2_n}{4mR}$ (b) $\dfrac{D_{n+m} - D_n}{4mR}$ (c) $\dfrac{4mR}{D_{n+m} - D_n}$ (d) $\dfrac{4mR}{D^2_{n+m} - D^2_n}$

21. D_n and D_{n+m} are the diameters of n^{th} and $(n + m)^{th}$ Newton's rings, when air is present between the lens and the plane glass plate. Also, D'_n and D'_{n+m} are the diameters of the n^{th} and $(n + m)^{th}$ Newton's rings, when a liquid of refractive index 'μ' is present between the lens and the plane glass plate. The expression for 'μ' is __________ .

(a) $\dfrac{D'^2_{n+m} - D'^2_n}{D^2_{n+m} - D^2_n}$ (b) $\dfrac{D^2_{n+m} - D^2_n}{D'^2_{n+m} - D'^2_n}$ (c) $\dfrac{D_{n+m} - D_n}{D'_{n+m} - D'_n}$ (d) $\dfrac{D'_{n+m} - D'_n}{D_{n+m} - D_n}$

22. Newton's rings are formed by the interference of light rays reflected from __________ .

(a) the lower surface of lens and the lower surface of glass plate
(b) the upper surface of lens and the upper surface of glass plate
(c) the lower surface of lens and the upper surface of glass plate
(d) the upper surface of lens and the lower surface of glass plate

Answers

1. a	2. d	3. b	4. c	5. b	6. a	7. d	8. b	9. a	10. a	11. b
12. a	13. c	14. a	15. a	16. b	17. d	18. d	19. c	20. a	21. b	22. c

Review Questions

1. Explain the concept of coherence. [Sept. 2009, Set 2]; [May 2008, Set 1]
2. Discuss why two independent sources of light of the same wave length cannot produce interference fringes. [Sept. 2008, Set 2]; [May 2008, Set 1]
3. What are the necessary conditions for obtaining interference fringes? [Sept. 2008, Set 2]; [May 2008, Set 1]
4. Define interference of light. [May 2008, Set 2]
5. Describe and explain the phenomenon of interference of light. [May 2008, Set 2]
6. Explain how Newton's rings are formed. [June 2009, Set 3]
7. How do you obtain circular rings in the Newton's rings experiment? [June 2009, Set 3]
8. Derive an expression for the radius of curvature in the Newton's ring experiment. [June 2009, Set 4]
9. Prove that the diameters of the bright rings are proportional to the square root of odd natural numbers. [June 2009, Set 4]
10. What do you mean by constructive and destructive interference of light? [Sept. 2008, Set 3]
11. Derive an expression for constructive maxima and destructive minima. [Sept. 2008, Set 3]

CHAPTER 2

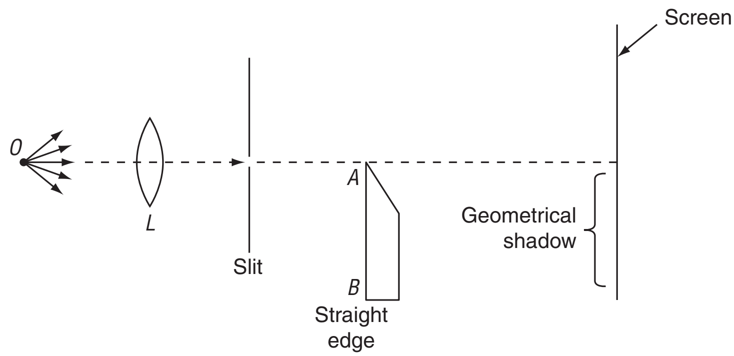
Diffraction

2.1 Introduction

In general, bending of waves round the edges of obstacles is known as diffraction. The amount of bending depends on the wave length of waves and the size of the obstacle. Bending is large in case of sound waves because of their longer wave length. Light waves also bend to small extent [because of shorter wave length] as they pass through the sharp edges of obstacles or narrow apertures. The bending of light waves is more pronounced when the size of the aperture is comparable with the wave length of light. This bending of light is illustrated in Fig. 2.1.

O is a monochromatic light placed at the focus of convex lens *L*. The parallel rays from *L* pass through the slit and then through a straight edge *AB*. Small amount of light bends around the edge of *AB* and is seen in the geometrical shadow of *AB* on the screen.

Figure 2.1 Diffraction at straight edge



Above the shadow, parallel to the edge (A) several bright and dark bands are seen due to diffraction. Thus the bending of light waves round the edges of opaque obstacles or narrow slits and spreading of light into geometrical shadow region is known as diffraction of light.

2.2 Fresnel and Fraunhofer diffraction

According to Fresnel, diffraction is due to the interference of the various secondary wave lets originated from the wave front which are not obstructed by the obstacle. The diffraction phenomena are divided into two classes, they are (i) Fresnel diffraction and (ii) Fraunhofer diffraction.

(i) Fresnel diffraction: In this class of diffraction, the source of light and the screen are at finite distance from the diffracting aperture or obstacle having sharp edge. The wave front incident on the aperture or obstacle is either spherical or cylindrical.

(ii) Fraunhofer diffraction: In this class of diffraction the source of light and the screen are at infinite distance from the diffraction aperture or obstacle having sharp edge. This can be achieved by placing the light source at the focal plane of the convex lens and placing the screen at the focal plane of another convex lens. In this case the wave front incident on the aperture or obstacle is a plane wave front.

Comparison between Fresnel and Fraunhofer diffractions

Fresnel Diffraction	Fraunhofer Diffraction
1. Point source of light or an illuminated narrow slit is used as light source	1. Extended source of light at infinite distance is used as light source
2. Light incident on the aperture or obstacle is a spherical or cylindrical wave front	2. Light incident on the aperture or obstacle is a plane wave front
3. The source and screen are at finite distance from the aperture or obstacle producing diffraction	3. The source and screen are at infinite distance from the aperture or obstacle producing diffraction
4. Lenses are not used to focus the rays	4. Converging lens is used to focus the rays

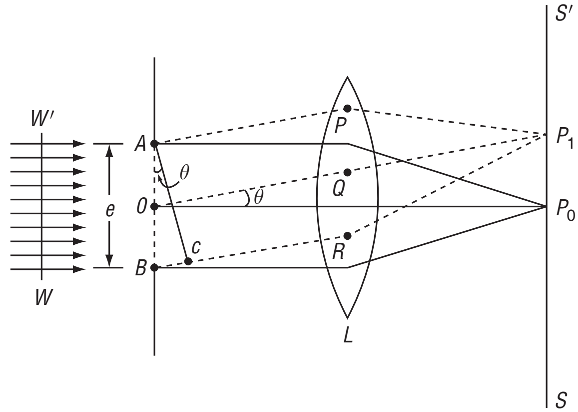
2.3 Fraunhofer diffraction at a single slit

The single slit is represented as AB in Fig. 2.2. The slit is in the form of a narrow rectangle in shape. The width of the slit AB is equal to 'e' and the plane of the slit is perpendicular to the plane of the paper. A plane wave front WW' of monochromatic light of wave length 'λ' is incident on the slit AB. Every point on the wave front in the slit will act as a source of secondary wavelets. The secondary wavelets travelling in the direction of OP_0 are brought to focus at P_0 on the screen SS' by using a converging lens L. The secondary wavelets from AB which are brought to focus at P_0 have no path difference. Hence the intensity at P_0 is high and it is known as central maximum. The secondary wavelets in the slit AB which make an angle 'θ' with OP_0 direction are brought to a point P_1 on the screen.

Intensity at point P_1 depends on the path difference between the wavelets at A and at B reaching to point P_1. To find the path difference, a perpendicular AC is drawn to BR from A. Now the path difference between the secondary wavelets from A and B in the direction of OP_1 is BC

$\therefore BC = AB \sin\theta = e \sin\theta$

Figure 2.2 Fraunhofer diffraction at a single slit



The corresponding phase difference is $(2\pi/\lambda)\, e \sin\theta$

Let us consider the width of the slit is divided into n equal parts. Then the phase difference between any two successive parts is $(1/n \times 2\pi/\lambda)\, e\sin\theta$. Let $(2\pi/n\lambda)\, e\sin\theta = d$ and let the amplitude of the wave in each part be a. The resultant amplitude R using the *vector addition* method is

$$R = a\frac{\sin nd/2}{\sin d/2} = a\frac{\sin\left((\pi/\lambda)\, e\sin\theta\right)}{\sin\left((\pi/n\lambda)\, e\sin\theta\right)}$$

Put $\alpha = (\pi/\lambda)\, e\sin\theta$

$$\text{then } R = a\frac{\sin\alpha}{\sin(\alpha/n)} \approx a\frac{\sin\alpha}{(\alpha/n)} \quad [\text{since } \alpha/n \text{ is very small}]$$

$$= na\frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha} \qquad \text{———— (2.1)}$$

where $A = na$

Equation (2.1) represents the resultant amplitude. Squaring Equation (2.1) gives the intensity (I) of light.

$$I = R^2 = A^2\left[\frac{\sin\alpha}{\alpha}\right]^2 \qquad \text{———— (2.2)}$$

Principal maximum: Equation (2.1) can be written in powers of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ldots\ldots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots\ldots\right] \qquad \text{———— (2.3)}$$

R will be maximum when $\alpha = 0$

$$\text{i.e. } \alpha = \frac{\pi}{\lambda} e \sin\theta = 0 \Rightarrow \sin\theta = 0 \quad \text{or} \quad \theta = 0$$

Hence the secondary wavelets that travel normal to the slit can produce maximum amplitude known as principal maximum.

Minimum intensity positions: The intensity will be minimum when sin α in Equation (2.2) is zero i.e., $\sin \alpha = 0$

The values of α are $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots = \pm m\pi$ or $\frac{\pi}{\lambda} e\sin\theta = \pm m\pi$

$$\text{or } e\sin\theta = \pm m\lambda \qquad \text{\_\_\_\_\_\_\_\_\_\_ (2.4)}$$

where $m = 1, 2, 3, \ldots$

$m = 0$ corresponds to principal maximum

From Equation (2.4) we know that the position of minimum intensity are found on both sides of principal maximum.

Secondary maxima: Between equally spaced minima weak secondary maxima are found. The positions of secondary maxima can be obtained by differentiating the intensity of light (I) with 'α' and equating it to zero.

From Equation (2.2)

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$\text{i.e. } A^2 . \frac{2\sin\alpha}{\alpha} . \left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

In the above equation either $\sin \alpha = 0$ or

$$\alpha \cos \alpha - \sin \alpha = 0$$

We already come across $\sin \alpha = 0$ for minimum intensity positions. Hence $\alpha\cos \alpha - \sin \alpha = 0$

$$\text{or } \alpha = \tan \alpha \qquad \text{\_\_\_\_\_\_\_\_\_\_ (2.5)}$$

The values of α that satisfy the above equation can be obtained by plotting $y = \alpha$ and $y = \tan \alpha$ on the same graph (Fig. 2.3). The points of intersection of two curves give the value of 'α' which satisfy Equation (2.5).

The points of intersection are $\alpha = 0, \pm 3\pi/2, \pm 5\pi/2,$ etc. Substituting these values in Equation (2.2), we get the intensities of various maxima.

For $\alpha = 0$ $\quad I_0 = A^2$ [Principal maximum]

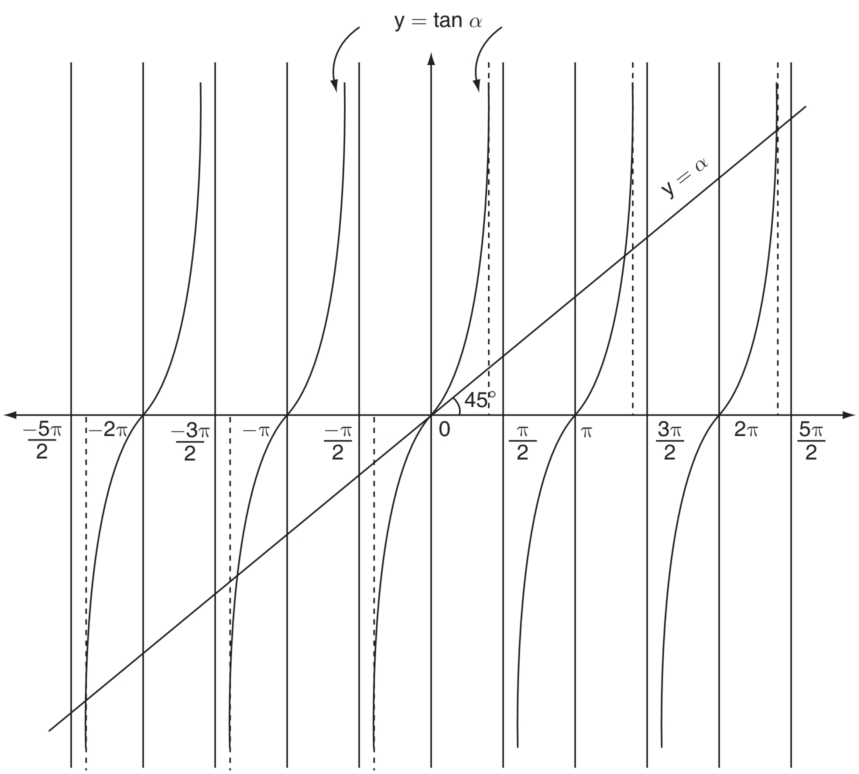
$$\text{For } \alpha = \frac{3\pi}{2} \qquad I_1 = A^2\left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{A^2}{22} \text{ (approximately)}$$

$$\text{For } \alpha = \frac{5\pi}{2} \qquad I_2 = A^2\left[\frac{\sin(5\pi/2)}{(5\pi/2)}\right]^2 = \frac{A^2}{62} \text{ (approximately)}$$

and so on

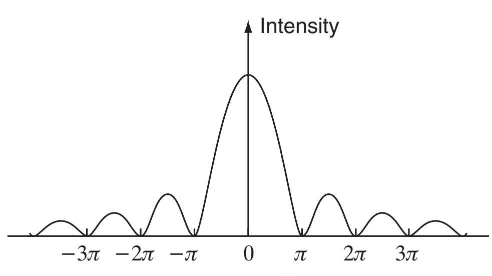
From the above values, we come to know that most of the incident light is found in the principal maximum.

Figure 2.3 $y = \alpha$ and $y = \tan \alpha$ graph $\left[\alpha = \pi/\lambda\, e\sin\theta\right]$ for single slit diffraction



A graph is drawn between intensity of light versus α (Fig. 2.4). The principal maximum occurs at the centre of diffraction pattern and the subsidiary maxima of decreasing intensity on both sides of principal maximum. The subsidiary maxima are not exactly at the centres of minima but slightly towards the centre of the diffraction pattern.

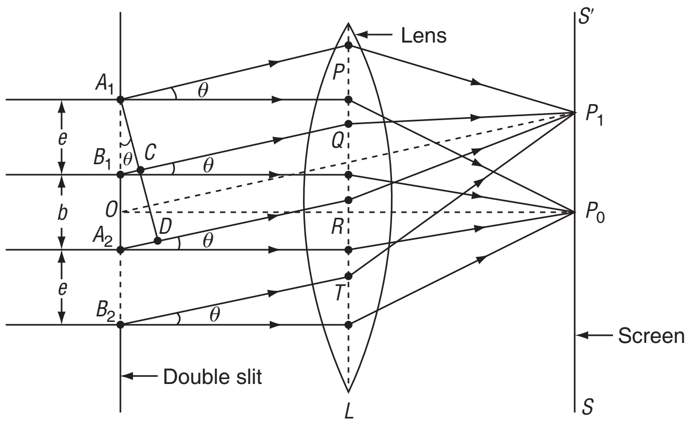
Figure 2.4 Intensity distribution due to diffraction at a single slit



2.4 Fraunhofer diffraction at double slit

The double slits have been represented as A_1B_1 and A_2B_2 in Fig. 2.5. The slits are narrow and rectangular in shape. The plane of the slits is perpendicular to the plane of the paper. Let the width of both the slits be equal and it is 'e' and they are separated by opaque length 'b'. A monochromatic plane wave front of wave length 'λ' is incident normally on both the slits.

Figure 2.5 Fraunhofer diffraction at double slits



Theory: Every point on the wave front in the slits will act as a source of secondary wavelets. The secondary wavelets travelling in the direction of OP_0 are brought to focus at P_0 on the screen SS' by using a converging lens L. P_0 corresponds to the position of the central bright maximum. The intensity distribution on the screen is the combined effect of interference of diffracted secondary waves from the slits.

The diffracted intensity on the screen is very large along the direction of incident beam [i.e. along OP_0]. Hence it is maximum at P_0. This is known as principal maximum of zero order.

The intensity at point P_1 on the screen is obtained by applying the Fraunhofer diffraction theory at single slit and interference of diffracted waves from the two slits. The diffracted wave amplitude due to single slit at an angle θ with respect to incident beam is $A(\sin\alpha/\alpha)$, where 2α is the phase difference between the secondary wavelets arising at the end points of a slit. This phase difference can be estimated as follows: Draw a normal from A_1 to B_1Q. Now, B_1C is the path difference between the diffracted waves at an angle 'θ' at the slit A_1B_1.

From the triangle A_1B_1C

$$\sin\theta = \frac{B_1C}{A_1B_1} = \frac{B_1C}{e} \quad \text{or} \quad B_1C = e\sin\theta$$

The corresponding phase difference $(2\alpha) = (2\pi/\lambda)\, e\sin\theta$

$$\text{or} \qquad \alpha = \frac{\pi}{\lambda} e\sin\theta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \quad (2.6)$$

The diffracted wave amplitudes, $[A(\sin\alpha/\alpha)]$ at the two slits combine to produce interference. The path difference between the rays coming from corresponding points in the slits A_1B_1 and A_2B_2 can be found by

drawing a normal from A_1 to A_2R. A_2D is the path difference between the waves from corresponding points of the slits.

In the triangle A_1A_2D $\frac{A_2D}{A_1A_2} = \sin\theta$ or the path difference $A_2D = A_1A_2 \sin\theta = (e + b)\sin\theta$

The corresponding phase difference $(2\beta) = \frac{2\pi}{\lambda}(e+b)\sin\theta$ ___________ (2.7)

Applying the theory of interference on the wave amplitudes $[A(\sin\alpha/\alpha)]$ at the two slits gives the resultant wave amplitude (R).

$$R = 2A\frac{\sin\alpha}{\alpha}\cos\beta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.8)}$$

The intensity at P_1 is

$$I = R^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

$$= 4I_o\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.9)} \qquad [\text{since } I_o = A^2]$$

Equation (2.9) represents the intensity distribution on the screen. The intensity at any point on the screen depends on α and β. The intensity of central maximum is $4I_o$. The intensity distribution at different points on the screen can be explained in terms of path difference between the incident and diffracted rays as follows. In Equation (2.9) the term $\cos^2\beta$ corresponds to interference and $\sin^2\alpha/\alpha^2$ corresponds to diffraction. Now, we shall look at the conditions for interference and diffraction maxima and minima.

Interference maxima and minima: If the path difference $A_2D = (e + b)\sin\theta_n = \pm n\lambda$ where n = 1, 2, 3… then 'θ_n' gives the directions of the maxima due to interference of light waves coming from the two slits. The ± sign indicates maxima on both sides with respect to the central maximum. On the other hand if the path difference is odd multiples of $\lambda/2$ i.e., $A_2D = (e+b)\sin\theta_n = \pm(2n-1)\lambda/2$, then θ_n gives the directions of minima due to interference of the secondary waves from the two slits on both sides with respect to central maximum.

Diffraction maxima and minima: If the path difference $B_1C = e\sin\theta n = \pm n\lambda$, where n = 1, 2, 3… then θ_n gives the directions of diffraction minima. The ± sign indicates minima on both sides with respect to central maximum. For diffraction maxima $e\sin\theta_n = \pm(2n-1)\lambda/2$ is the condition. The ± sign indicates maxima on both sides with respect to central maximum.

The intensity distribution on the screen due to double slit diffraction is shown in Fig. 2.12. Fig. 2.6(a) represents the graph for interference term, Fig. 2.6(b) shows the graph for diffraction term and Fig 6(c) represents the resultant distribution.

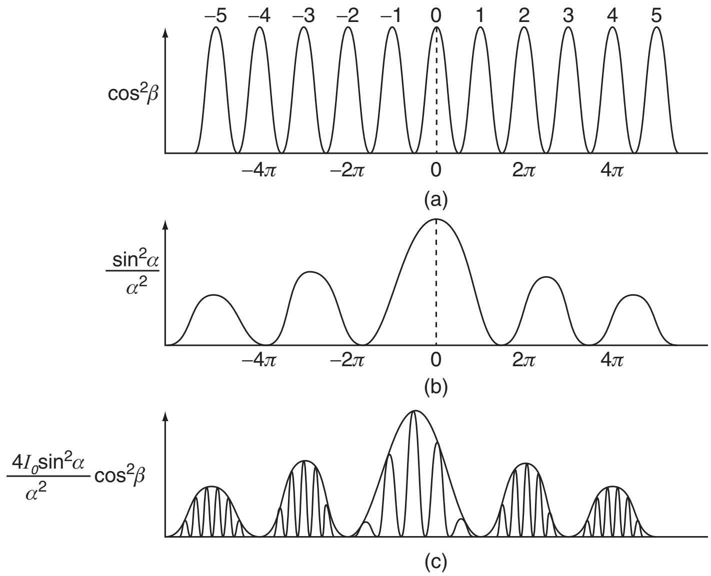
Missing orders in a double slit diffraction pattern: Based on the relative values of e and b certain orders of interference maxima are missing in the resultant pattern.

Based on the relative values of e and b certain orders of interference maxima are missing in the resultant pattern.

The direction of interference maxima are given as $(e + b)\sin\theta_n = n\lambda$ where $n = 1, 2, 3, \ldots$.. and the directions of diffraction minima are given as $e\sin\theta_m = m\lambda$ where $m = 1, 2, 3, \ldots$

For some values of θ_n, the values of e and b are satisfied such that at these positions the interference maxima and the diffraction minima are formed. The combined effect results in missing of certain orders of interference maxima. Now we see certain values of e and b for which interference maxima are missing.

Figure 2.6 Intensity distribution due to diffraction at double slit



(i) Let $e = b$

Then, $2\,e\sin\theta_n = n\lambda$ and $e\sin\theta_m = m\lambda$

$$\therefore \frac{n}{m} = 2 \quad \text{or} \quad n = 2m$$

If $m = 1, 2, 3\ldots$ then $n = 2, 4, 6\ldots$, i.e. the interference orders 2, 4, 6 … missed in the diffraction pattern

(ii) If $2e = b$

Then $3e\sin\theta_n = n\lambda$ and $e\sin\theta_m = m\lambda$

$$\therefore \frac{n}{m} = 3 \quad \text{or} \quad n = 3m$$

if $m = 1, 2, 3\ldots$ Then $n = 3, 6, 9\ldots$, i.e. the interference orders 3, 6, 9… are missed in the diffraction pattern

(iii) if $e + b = e$

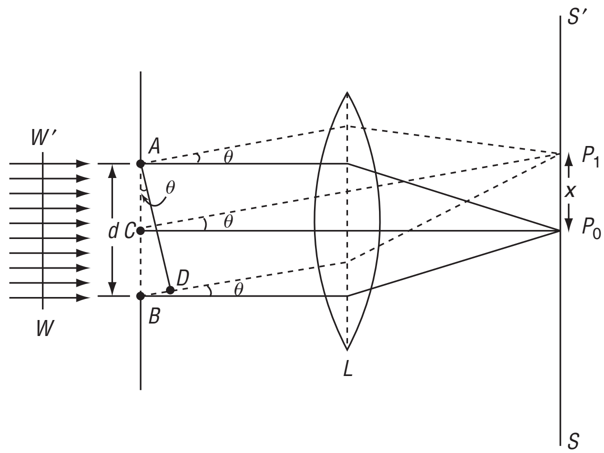
i.e. $b = 0$

the two slits are joined. So, the diffraction pattern is due to a single slit of width $2e$.

2.5 Fraunhofer diffraction at a circular aperture

The problem of diffraction at a circular aperture was first solved by Airy in 1835. A circular aperture of diameter 'd' is shown as AB in Fig. 2.7. A plane wave front WW' is incident normally on this aperture. Every point on the plane wave front in the aperture acts as a source of secondary wavelets. The secondary wavelets spread out in all directions as diffracted rays in the aperture. These diffracted secondary wavelets are converged on the screen SS' by keeping a convex lens (L) between the aperture and the screen. The screen is at the focal plane of the convex lens. Those diffracted rays traveling normal to the plane of aperture [i.e., along CP_o] are get converged at P_o.

Figure 2.7 Fraunhofer diffraction at a circular aperture



All these waves travel the same distance to reach P_o and there is no path difference between these rays. Hence a bright spot is formed at P_o known as Airy's disc. P_o corresponds to the central maximum.

Next consider the secondary waves traveling at an angle θ with respect to the direction of CP_o. All these secondary waves travel in the form of a cone and hence, they form a diffracted ring on the screen. The radius of that ring is x and its centre is at P_o. Now consider a point P_1 on the ring, the intensity of light at P_1 depends on the path difference between the waves at A and B to reach P_1. The path difference is $BD = AB \sin \theta = d \sin \theta$. The diffraction due to a circular aperture is similar to the diffraction due to a single slit. Hence, the intensity at P_1 depends on the path difference $d \sin \theta$. If the path difference is an integral multiple of λ then intensity at P_1 is minimum. On the other hand, if the path difference is in odd multiples of $\lambda/2$, then the intensity is maximum.

i.e. $d \sin \theta = n\lambda$ for minima ___________ (2.10)

and $d \sin \theta = (2n-1)\dfrac{\lambda}{2}$ for maxima ___________ (2.11)

where $n = 1, 2, 3, \ldots.$ etc. $n = 0$ corresponds to central maximum.

The Airy disc is surrounded by alternate bright and dark concentric rings, called the Airy's rings. The intensity of the dark ring is zero and the intensity of the bright ring decreases as we go radially from P_0 on the screen. If the collecting lens (L) is very near to the circular aperture or the screen is at a large distance from the lens, then

$$\sin\theta \approx \theta \approx \frac{x}{f} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_ (2.12)}$$

Where f is the focal length of the lens.
Also from the condition for first secondary minimum [using Equation (2.10)]

$$\sin\theta \approx \theta \approx \frac{\lambda}{d} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_ (2.13)}$$

Equations (2.12) and (2.13) are equal

$$\text{So, } \frac{x}{f} = \frac{\lambda}{d} \quad \text{or} \quad x = \frac{f\lambda}{d} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_ (2.14)}$$

But according to Airy, the exact value of x is

$$x = \frac{1.22 f \lambda}{d} \qquad \text{———} \qquad (2.15)$$

using Equation (2.15) the radius of Airy's disc can be obtained. Also from Equation (2.15) we know that the radius of Airy's disc is inversely proportional to the diameter of the aperture. Hence by decreasing the diameter of aperture, the size of Airy's disc increases.

2.6 Plane diffraction grating [Diffraction at *n* slits]

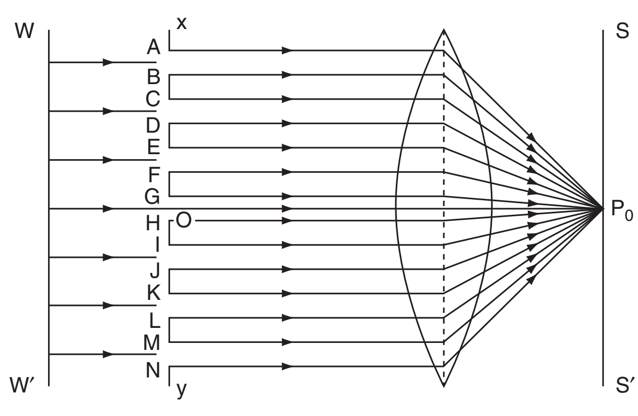
A large number of narrow rectangular slits having width in the order of the wave length of light and arranged side by side with equal opaque spaces is called a plane transmission diffraction grating. It is an extension of diffraction due to double slit. The first diffraction grating constructed by Fraunhofer consists of a large number of parallel wires of 0.05 mm diameter and they are separated by 0.0533 mm to 0.687 mm. A plane transmission grating can be easily constructed by drawing a large number of closely spaced lines on a plane transparent plate like glass with a sharp diamond point. The lines on the plate are opaque to light and the spaces between these lines are transparent. Usually 15,000 lines are drawn on one inch width of the grating. Similar to plane transmission grating, plane reflection grating can be constructed by drawing closely spaced lines on the silvered surface of a plane mirror.

Theory of transmission grating:

In Fig. 2.8, XY represents the grating. The plane of the grating is perpendicular to the plane of the paper. In the grating AB, CD, EF, etc. represents the slits, each of width 'e' and these slits are separated by equal opaque regions BC, DE, FG, etc. each of width 'b'. $a + b$ is called the grating element and if any two points in the consecutive slits are separated by $a + b$ then the points are called corresponding points.

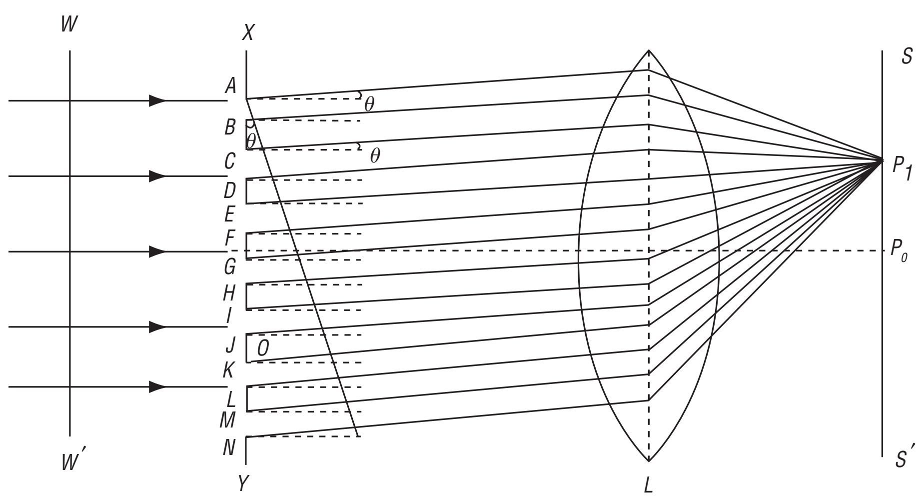
Let a monochromatic plane wave front WW' of wave length λ be incident on the slits. Each point on the wave front in the slits will act as a source of secondary wavelets. These wavelets will travel in all

Figure 2.8 Diffraction grating



directions. The secondary wavelets in each slit will produce diffraction. The diffracted secondary wavelets of all the slits combine to produce interference. Now all the secondary wavelets travelling in the direction of the incident beam will get converged at P_o on the screen SS' by passing through a convex lens L having focal length f. The screen is at the focal plane of the lens. The intensity at P_o is very high and it is called as central maximum. Next we see the secondary wavelets travelling in a direction that makes an angle θ with the direction of incident beam. As shown in Fig. 2.9 the secondary wavelets which make an angle θ will reach the point P_1 on the screen.

Figure 2.9 **Diffraction due to grating showing intensity at P_1**



The intensity at P_1 can be obtained by considering diffraction at each slit and interference of all these diffracted waves at an angle θ with the incident beam. The resultant amplitude due to diffraction at single slit is $A\ (\sin\alpha/\alpha)$, which is now at the middle point of each slit where, $\alpha = (\pi/\lambda)\, e\sin\theta$. Let us assume that there are N slits on the grating. The path difference between the corresponding points is $(e + b)\sin\theta$ and in terms of phase difference it is $2\pi/\lambda(e+b)\ \sin\theta = 2\beta$. The resultant of N amplitudes, each of $A(\sin\alpha/\alpha)$ with common phase difference (2β) between consecutive diffracted amplitudes can be obtained by the vector addition method. The resultant amplitude (R) of the N amplitudes each of $A(\sin\alpha/\alpha)$ is

$$R = \frac{A\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta} \qquad \text{————— (2.16)}$$

and intensity

$$I = R^2 = \left(\frac{A\ \sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \qquad \text{————— (2.17)}$$

The value $(A\ \sin\alpha/\alpha)^2$ shows the intensity distribution due to single slit diffraction and $\left(\sin^2 N\beta/\sin^2\beta\right)$ shows the combined intensity distribution due to interference at all the N slits.

Principal maxima: From Equation (2.17), we know that the intensity would be maximum when $\sin\beta = 0$, where $\beta = 0, \pi, 2\pi, 3\pi \ldots = n\pi$ and $n = 0, 1, 2, 3 \ldots$

Then $\dfrac{\sin N\beta}{\sin\beta} = \dfrac{0}{0}$

The above factor is indeterminate: To find its value, the numerator and denominator are differentiated separately. i.e., L'Hospital's rule is applied.

$$\underset{\beta\to\pm n\pi}{\mathrm{Lt}} \frac{\sin N\beta}{\sin\beta} = \underset{\beta\to\pm n\pi}{\mathrm{Lt}} \frac{\dfrac{d}{d\beta}\sin(N\beta)}{\dfrac{d}{d\beta}\sin\beta}$$

$$= \underset{\beta\to\pm n\pi}{\mathrm{Lt}} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

Hence, the resultant intensity [from equation (2.17)] is

$$I = N^2\left[\frac{A\sin\alpha}{\alpha}\right]^2 = N^2 I_o \frac{\sin^2\alpha}{\alpha^2} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.18)}$$

These maxima are most intense and are called principal maxima. The maxima are obtained for $\beta = \pm n\pi$ or

$$\frac{\pi}{\lambda}(e+b)\sin\theta = \pm n\lambda \quad \text{or}$$

$$(e+b)\sin\theta = \pm n\lambda \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.19)}$$

$n = 0$ corresponds to zero order (or central) maximum. $n = 1, 2, 3 \ldots$ corresponds to the first, second, third, etc. principal maxima, respectively. The $\pm$ sign shows that the principal maxima are present on both sides of the central maximum.

Minima: From Equation (2.17) we know a series of minima occur when $\sin N\beta = 0$ but $\sin\beta \neq 0$.

$$\text{So } N\beta = \pm m\pi \quad \text{or} \quad \frac{N\pi}{\lambda}(e+b)\sin\theta = \pm m\pi \quad \text{or}$$

$N(e+b)\sin\theta = \pm m\lambda$. m can take integral values except $0, N, 2N, \ldots n$N. For these values we get maxima

Secondary maxima: Between two adjacent principal maxima there are $(N-1)$ minima and $(N-2)$ other maxima called secondary maxima. To get the positions of secondary maxima, differentiate Equation (2.17) w.r.t. β and then equate it to zero.

$$\text{Thus } \frac{dI}{d\beta} = \left(\frac{A\sin\alpha}{\alpha}\right)^2 \cdot 2\left(\frac{\sin N\beta}{\sin\beta}\right) \times \left[\frac{\mathrm{N}\cos \mathrm{N}\beta\sin\beta - \sin \mathrm{N}\beta\cos\beta}{\sin^2\beta}\right] = 0$$

So

$$N\cos N\beta\sin\beta - \sin N\beta\cos\beta = 0 \quad \text{or}$$

$$N\tan\beta = \tan N\beta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.20)}$$

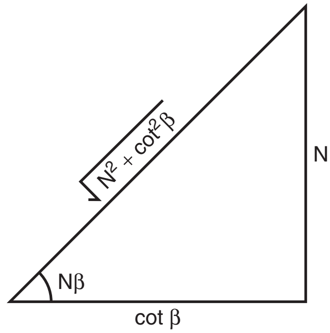
The roots of the above equation [other than $\beta = \pm n\pi$, this corresponds to principal maxima] give the positions of secondary maxima. To find the intensity $\sin^2 N\beta/\sin^2\beta$ from $N\tan\beta = \tan N\beta$, we use the triangle shown below.

$$\sin N\beta = \frac{N}{\sqrt{N^2 + \cot^2\beta}}$$

Squaring the above equation and dividing by $\sin^2\beta$, we have

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{\left(N^2+\cot^2\beta\right)\times\sin^2\beta}$$

$$= \frac{N^2}{N^2\sin^2\beta+\cos^2\beta} = \frac{N^2}{1+\left(N^2-1\right)\sin^2\beta}$$



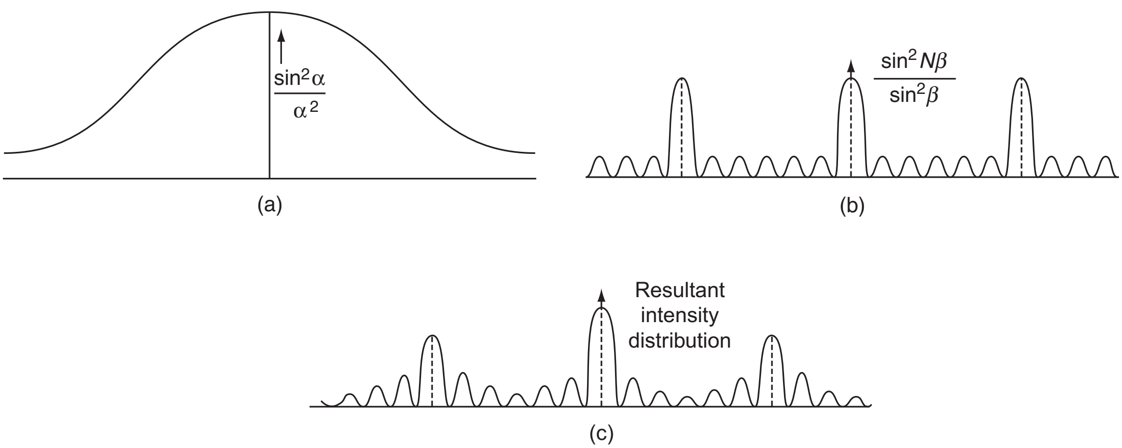
Now,

$$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{1}{1+\left(N^2-1\right)\sin^2\beta}$$

From the above equation, we know that as N increases the intensity of secondary maxima relative to principal maxima decreases. When N is very large, the intensity of secondary maxima is very less compared to principal maxima.

The resultant intensity distribution on both sides of the central maximum can be represented by the diffraction term $\left(\sin^2\alpha/\alpha^2\right)$ and by the interference term $\sin^2 N\beta/\sin^2\beta$. The variation of the above two terms and the resultant from the central maximum are shown in Fig. 2.10.

Figure 2.10 **(a) Represents the variation of intensity due to the factor $\sin^2\alpha/\alpha^2$; (b) Represents the variation of intensity due to the factor $\sin^2 N\beta/\sin^2\beta$; (c) Represents the resultant variation of intensity**

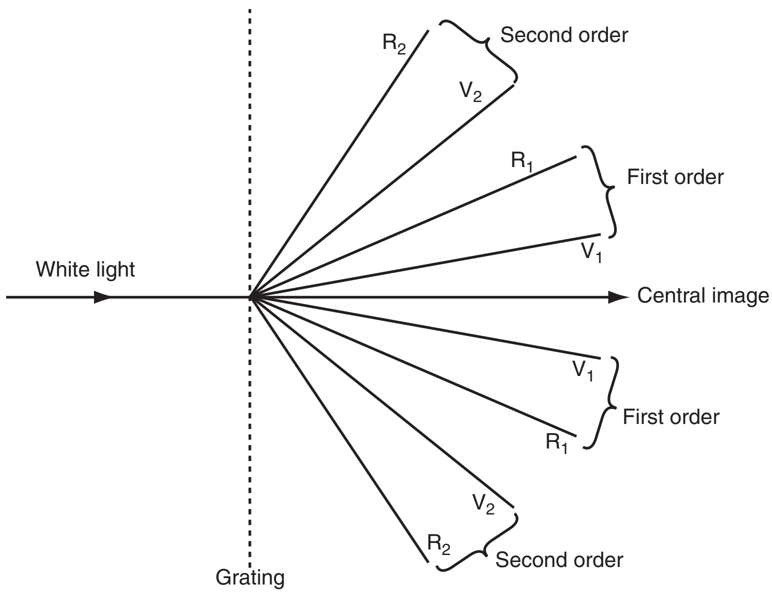


2.7 Grating spectrum

The diffraction pattern formed with a grating is known as a grating spectrum. From Equation (2.19), $(e + b)\sin\theta = \pm n\lambda$, $n = 0, 1, 2, 3, \ldots$ we know the θ values for different principal maxima. If the number of lines in the grating are very large then the maxima are sharp and bright. Lines parallel to the grating lines

are called spectral lines. Different order bright lines are formed on both sides of central maximum. Instead of monochromatic light if white light is used, then light of different wave lengths are diffracted in different directions for each order of diffraction as shown in Fig. 2.11.

Figure 2.11 Grating spectrum for white light source



The central image is white because the central maxima of different wave length rays coincide at that place. In each order of diffraction, waves having longer wave lengths are diffracted with greater diffraction angle.

Maximum number of orders available with a grating

From Equation (2.19), $(e \pm b)\ \sin\theta = n\lambda$. We have

$$n = \frac{(e+b)\sin\theta}{\lambda}$$

The maximum angle of diffraction is 90°, hence the maximum order of diffraction n_{max} is

$$n_{max} \leq \frac{(e+b)\sin 90^\circ}{\lambda} \leq \frac{(e+b)}{\lambda} \leq \frac{1}{N\lambda} \quad \text{where}$$

$$N = \frac{1}{e+b} = \text{Number of lines per unit distance of grating.}$$

$$\therefore \quad n_{max} \leq \frac{1}{N\lambda}$$

Determination of wave length of light using grating:

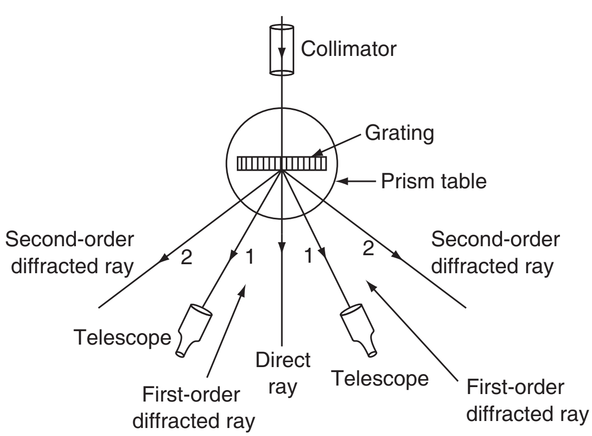
The wave length of a given source of light [monochromatic or polychromatic] can be determined using plane diffraction grating and spectrometer. For this, the following spectrometer adjustments are to be carried out.

1 The telescope is focused to a distant object and adjusted to see a clear image of the distant object.
2. The telescope and collimator are brought in line.
3. The slit of the collimator is illuminated with the given source of light.
4. Seeing through the telescope the slit of the collimator is adjusted to see a thin clear source of light.
5. The telescope is moved so that the vertical cross wire of the telescope coincides with the image of the slit. Then the telescope is rotated through 90° so that the collimator and telescope are perpendicular.
6. The diffraction grating is mounted vertically on the prism table and then rotated in the forward and backward directions so that the light from the collimator is reflected on the surface of the grating and the reflected light coincides with the vertical crosswire of the telescope. Then, fix the grating. Now the plane of the grating makes 45° with the incident beam.
7. Release the base of spectrometer and rotate the grating through 45° so that the surface of the grating is normal to the incident beam.

Now the telescope is released and brought inline with the collimator to view the direct ray. Seeing through the telescope, it is moved slowly to one side until the first order diffracted spectral line is seen as shown in Fig. 2.12. The vertical cross wire of the telescope is made to coincide with the first order spectral line and note down the spectrometer reading. Again, bring the telescope in line with the collimator and move it to other side to see the first order spectral line on the other side. Coincide the vertical cross wire of telescope with this spectral line and take the reading. The difference between these two readings gives twice the diffraction angle (2θ). Using the formula $\sin \theta = n N\lambda$ [n = order of diffracted ray, N = number of lines per unit width of grating, λ = wave length of monochromatic light] the wave length of monochromatic light can be determined. The same process can be repeated for second order and even for higher orders.

In case of white light, the diffraction angles are measured for different colours and hence the different wave lengths are determined.

Figure 2.12 Determination of wave length of monochromatic light using spectrometer



2.8 Rayleigh's criterion for resolving power

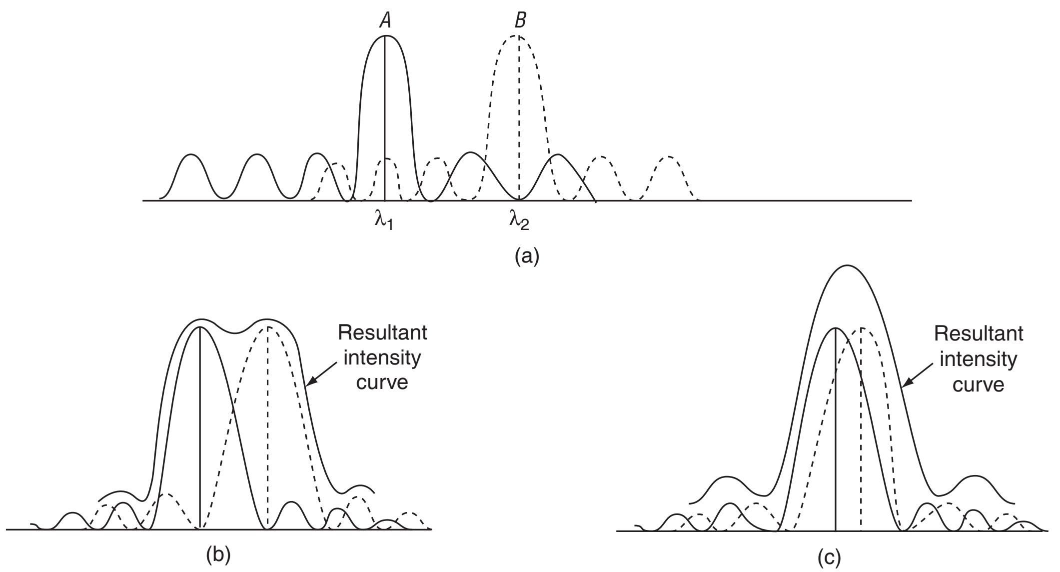
According to Rayleigh's criterion, two nearby images are said to be resolved if they are separated by at least a certain minimum distance so that the position of diffraction central maximum of the first image coincides with the first diffracted minimum of the second image in any order of diffraction and vice versa.

Rayleigh's criterion can be explained by considering the diffraction patterns due to two spectral lines of wave lengths λ_1 and λ_2. Let *A* and *B* represent central maxima of diffraction patterns corresponding to wave lengths λ_1 and λ_2. As shown in fig 13(a), the diffraction angle for the central maximum of the image *B* is larger than the diffraction angle for the central minimum of the image *A*. The principal maximum of *A* is far away from the first minimum of *B* and vice versa. Hence, their principal maxima are separately visible. The intensity between these two principal maxima is zero. So, the two spectral lines corresponding to the two wave lengths are well resolved.

As shown in Fig. 13(b), let the central maximum of A coincide with the first minimum of *B* and vice versa. The resultant intensity is shown by a thick curve with a small dip in between the two central maxima. There is a small noticeable decrease in intensity between the two central maxima. This indicates the presence of two wave lengths. Thus according to Rayleigh's condition, the spectral lines can be just resolved.

Lastly as shown in Fig. 13(c) when the diffracted central maxima of the two wave lengths are still closer, then the resultant intensity curve has no dip in the middle. The resultant intensity peak is higher than the individual intensities. Thus the two images overlap and they cannot be distinguished as separate images. So one cannot resolve them as two spectral lines.

Figure 2.13 Central maxima of diffraction patterns corresponding to wave length λ_1 and λ_2



2.9 Resolving power of a plane transmission grating

The capacity of an optical instrument to show separate images of very closely placed two objects is called resolving power. The resolving power of a diffraction grating is defined as its ability to form separate diffraction maxima of two closely separated wave lengths.

The expression for resolving power of a grating can be obtained with the aid of Fig. 2.14.

In Fig. 2.14, *XY* represents a plane transmission grating having $(e + b)$ as grating element and N lines per unit width of grating. A light beam having two slightly different wave lengths λ and $\lambda + d\lambda$ is incident normally on the surface of the grating. SS_1 is the screen, on the screen P_1 represents the position of n^{th} order primary maximum spectral line of wave length λ diffracted at an angle θ_n. Similarly P_2 represents the position of n^{th} order primary maximum spectral line of wave length $\lambda + d\lambda$, diffracted at an angle $\theta_n + d\theta$. These n^{th} order maxima lines can be resolved if P_2 corresponds to the first minimum of wave length λ.

The principal maximum of wave length λ at diffraction angle θ_n is $(e + b) \sin \theta_n = n\lambda$ ________ (2.21)

The equation for the first minimum of wave length λ in the direction $(\theta_n + d\theta)$ is

$N(e + b) \sin (\theta_n + d\theta) = (n N + 1) \lambda$ __________ (2.22)

Next, the n^{th} principal maximum for wave length $(\lambda + d\lambda)$ in the direction $\theta_n + d\theta$ is

$(e + b) \sin (\theta_n + d\theta) = n (\lambda + d\lambda)$ __________(2.23) Multiplying Equation (2.23) by N, we have

$N(e + b) \sin (\theta_n + d\theta) = n N(\lambda + d\lambda)$ __________ (2.24)

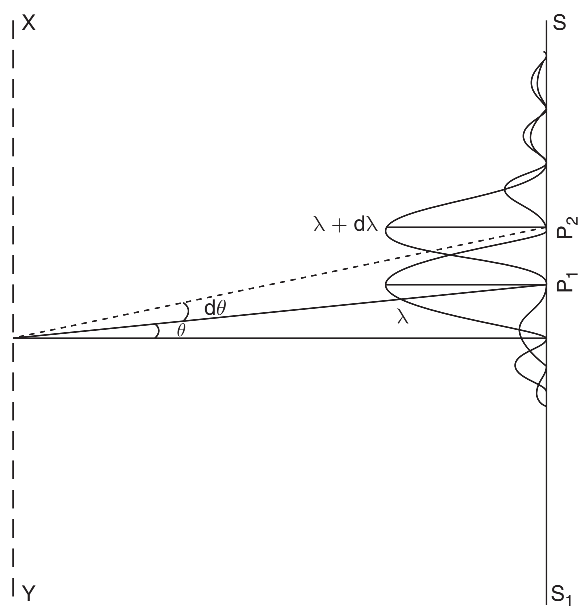
The LHS of Equations (2.22) and (2.24) are the same

$\therefore (n N+1)\lambda = nN(\lambda+d\lambda)$

or $\quad n N\lambda + \lambda = n N\lambda + n Nd\lambda$

or $\quad \lambda = n Nd\lambda \quad$ or $\quad \dfrac{\lambda}{d\lambda} = nN$ __________ (2.25)

Figure 2.14 **Calculation of resolving power of a plane transmission grating**



Equation (2.25) represents the expression for the resolving power of the grating. From Equation (2.25) we know that the resolving power is directly proportional to (i) the diffraction order of the spectrum and (ii) the total number of lines per unit width of grating.

Formulae

1. At the single slit:

$$R = A\frac{\sin\alpha}{\alpha} \quad \text{where} \quad A = na \quad \text{and} \quad \alpha = \frac{\pi}{\lambda}e\sin\theta$$

2. $$I = R^2 = A^2\frac{\sin^2\alpha}{\alpha^2}$$

3. Principal maximum

$$R = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right]$$

R is maximum when $\alpha = 0$, i.e. $\theta = 0$.

4. Condition for minimum intensity position is

$$e\sin\theta = \pm m\lambda \quad \text{where} \quad m = 1, 2, 3, \ldots$$

5. Secondary maxima condition $\alpha = \tan\alpha$ except for $\alpha = 0$, the secondary maxima positions are:

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \cdots$$

For $\alpha = 0, I_o = R^2$

For $$\alpha = \frac{3\pi}{2}, I_1 = A^2\left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{A^2}{22}$$

For $$\alpha = \frac{5\pi}{2}, I_2 = A^2\left[\frac{\sin(5\pi/2)}{(5\pi/2)}\right]^2 = \frac{A^2}{62}$$

At the double slit, the diffraction resultant (R) and intensity (I) are:

6. $$R = 2A\left(\frac{\sin\alpha}{\alpha}\right)\cos\beta, \text{ where } 2\beta = \frac{2\pi}{\lambda}(e+b)\sin\theta$$

7. $$I = R^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta = 4I_o\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

8. $(e + b)\sin\theta_n = \pm n\lambda$, where $n = 1, 2, 3, \ldots$ interference maxima

9. $(e + b)\sin\theta_n = \pm(2n-1)\frac{\lambda}{2}$, interference minima

10. $e\sin\theta_n = \pm n\lambda$, $n = 1, 2, 3, \ldots$ diffraction minima

11. $e\sin\theta_n = \pm(2n-1)\frac{\lambda}{2} \cdots$ diffraction maxima

12. At the circular aperture:
$d\sin\theta = n\lambda$ for minima

13. $d\sin\theta = (2n-1)\frac{\lambda}{2}$ for maxima

14. $x = \frac{1.22 f\lambda}{d}$ = radius of Airy's disc

15. At the n-slit, the diffraction amplitude and intensity are:

$$R = A\frac{\sin\alpha}{\alpha}\frac{\sin N\beta}{\beta}$$

16. $$I = R^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

17. $$I = N^2 I_o\frac{\sin^2\alpha}{\alpha^2}$$

18. $N(e + b)\sin\theta = \pm m\lambda$; $m \neq 0, N, 2N, \ldots nN$ for minima

19. Roots of $N\tan\beta = \tan N\beta$ give positions of secondary maxima (other than $\beta = \pm n\pi$)

20. Principal maxima is $(e + b)\sin\theta = \pm n\lambda$

21. $$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{1}{1 + (N^2 - 1)\sin^2\beta}$$

22. Resolving power of grating, $\frac{\lambda}{d\lambda} = nN$

Solved Problems

1. *A single slit of width 4×10^{-8} mm is illuminated by a monochromatic light of wavelength 6000Å. Find the angular separation of the diffracted first-order minimum from the central maximum.*

Sol: The expression for minimum intensity due to single slit diffraction is

$e \sin \theta_m = m\lambda$, where $m = 1, 2, 3, \ldots$

For first-order diffraction $m = 1$, it is

$e \sin \theta_1 = 1 . \lambda$

width of slit, $e = 4 \times 10^{-3}$ mm $= 4 \times 10^{-6}$ m

wavelength of light, $\lambda = 6000$ Å $= 6000 \times 10^{-10}$ m

$$\sin \theta_1 = \frac{\lambda}{e} = \frac{6000 \times 10^{-10}}{4 \times 10^{-6}} = \frac{0.3}{2}$$

$$\theta_1 = \sin^{-1}\left(\frac{0.3}{2}\right) = 8^\circ\ 37'\ 37''$$

2. *A monochromatic light of wavelength 6000×10^{-8} cm is diffracted by a single slit kept at a distance of 100 cm from the screen. The first diffracted minimum appears at a distance of 1 mm from the central maximum. Find the width of the slit.*

Sol: Wavelength of light, $\lambda = 6000 \times 10^{-8}$ cm $= 600 \times 10^{-10}$ m

The distance to the first minimum, $d = 1$ mm $= 1 \times 10^{-3}$ m

Distance of separation between slit and screen, $D = 100$ cm $= 1$ m

$$\text{Hence, } \sin \theta_1 \approx \frac{d}{D} = \frac{10^{-3}\text{m}}{1\text{m}} = 10^{-3}$$

we know $e \sin \theta_1 = 1\lambda$

$$e = \frac{\lambda}{\sin \theta_1} = \frac{6000 \times 10^{-10}\text{m}}{10^{-3}} = 6 \times 10^{-4}\text{m}$$

3. *A monochromatic light of wavelength 5500 Å is incident on a single slit of width 0.3 mm and gets diffracted. Find the diffraction angles for the first minimum and the next maximum.*

Sol: Wave length of light, $\lambda = 5500$Å $= 5500 \times 10^{-10}$ m

width of the slit, $e = 0.3$ mm $= 0.3 \times 10^{-3}$ m

The diffraction angle for the first minimum is given using the formula $e \sin \theta_n = n\lambda$

$$\text{i.e. } \sin \theta_1 = \frac{\lambda}{e}$$

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{e}\right) = \sin^{-1}\left[\frac{5500 \times 10^{-10}}{0.3 \times 10^{-3}}\right] = \sin^{-1}\left(\frac{0.55 \times 10^{-3}}{0.3}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{0.00055}{0.3}\right) = 0^\circ\ 6'\ 18''$$

For the first maximum from the central maximum, the formula is

$$e \sin \theta_1' = \frac{3\lambda}{2}$$

$$\theta_1' = \sin^{-1}\frac{3\lambda}{2e} = \sin^{-1}\left[\frac{3 \times 5500 \times 10^{-10}}{2 \times 0.3 \times 10^{-3}}\right] = \sin^{-1} 0.0028$$

$$= 0^\circ\ 9'\ 27''$$

4. *A circular aperture of diameter 0.1 mm is illuminated with sodium monochromatic light of wavelength 5893Å. The distance between the screen and lens is 1 m. Find the separation between the central disc and the first secondary minimum.*

Sol: The diameter of the circular aperture, d = 0.1 mm = 1×10^{-4} m

Wavelength of sodium light, λ = 5893 Å = 5893×10^{-10}m

The distance between screen and lens, f = 1 m

The radius of Airy's disc $x = \dfrac{1.22 f\lambda}{d} = \dfrac{1.22 \times 1 \times 5893 \times 10^{-10}}{1 \times 10^{-4}}$

$= 0.7189$ cm

5. *Mercury light is normally incident on a grating. The diffraction angle in the first-order spectrum for a green spectral line of wavelength 5460Å is 20°. Find the number of lines per cm of grating.*

Sol: The diffraction angle for green line, θ_g = 20°

Wavelength of green light, λ_g = 5460 Å = 5460×10^{-10}m

Order of diffraction, m = 1

No. of lines per cm, N = ?

Formula is $nN\lambda = \sin\theta$

$$N = \frac{\sin\theta}{n\lambda} = \frac{\sin 20°}{1 \times 5460 \times 10^{-10}} = 6,26,373.6 \text{ lines / m}$$

$$= 6263.7 \text{ lines / cm}$$

6. *A monochromatic light of wavelength 6560 Å is normally incident on a plane diffraction grating. The first-order spectral line is obtained at an angle of 18° 14′. Find the number of lines per cm on the grating.*

Sol: Wavelength of monochromatic light, λ = 6560 Å = 6560×10^{-10} m

First order diffraction, n = 1

Angle of diffraction, θ = 18° 14′

Number of lines per cm = ?

Formula is $n N \lambda = \sin\theta$

$$N = \frac{\sin\theta}{n\lambda} = \frac{\sin 18° \, 14'}{1 \times 6560 \times 10^{-10}} m = 476981.7 \text{ lines / m}$$

$$= 4769.817 \text{ lines / cm}$$

7. *There are 15000 lines per inch in a grating. What is the maximum number of orders obtained by using light of wavelength 6000 Å?*

Sol: Wavelength of light, λ = 6000 Å = 6000×10^{-10} m

Number of lines on grating, N = 15000 lines/inch $= \dfrac{15000}{2.54}$ lines / cm

$= \dfrac{1500000}{2.54}$ lines / m

Maximum number of orders, n_{max} = ?

Formula is

$$n N \lambda = \sin\theta$$

$$n_{max} N\lambda = \sin\theta_{max} = \sin 90° = 1$$

$$n_{max} = \frac{1}{N\lambda} = \frac{2.54}{1500000 \times 6000 \times 10^{-10}} = 2.82$$

$\therefore$ We can observe the first and second orders of diffraction only.

8. *Find the minimum number of lines required in a grating to resolve two spectral lines of wavelength 5890 Å and 5896 Å in second order diffraction.*

Sol: The average wavelength, $\lambda = \left(\frac{5890 + 5896}{2}\right)\mathring{A} = 5893\,\mathring{A}$

$= 5893 \times 10^{-10}\,\text{m}$

the difference in wavelengths,

$d\lambda = (5896 - 5890)\ \text{Å} = 6\ \text{Å}$

$= 6 \times 10^{-10}\text{m}$

Order of the spectrum, $n = 2$

Number of lines on the grating, $N = ?$

$$\frac{\lambda}{d\lambda} = nN$$

$$N = \frac{1}{n}\frac{\lambda}{d\lambda} = \frac{1}{2} \times \frac{5893 \times 10^{-10}}{6 \times 10^{-10}} = 491.08 \text{ lines}$$

9. *Find the resolving power of a grating having 6000 lines/cm in the first order diffraction. The rolled length of the grating is 15 cm.*

Sol: Lines per cm length of grating = 6000 lines/cm

Total number of lines on the grating, $N = 15 \times 6000 = 90{,}000$ lines

The formula is

$$\frac{\lambda}{d\lambda} = n\lambda = 1 \times 90{,}000 = 90{,}000$$

Multiple-choice Questions

1. Diffraction of light is __________.

(a) bending of waves around the edges of obstacles
(b) rectilinear propagation of light
(c) superposition of waves
(d) none of the above

2. In Fraunhofer class of diffraction, the source of light and screen are at ______ distance from the diffracting aperture or obstacle.

(a) finite (b) infinite (c) large (d) any

3. In Fresnel class of diffraction, the source of light and screen are at _____ distance from the diffracting aperture or obstacle.

(a) finite (b) infinite (c) large (d) any

4. In single slit diffraction, if e is the width of the slit, λ is the wavelength of light used and θ is the diffraction angle, then the condition for minimum intensity is ($m = 1, 2, 3, \ldots$) __________.

(a) $e \sin\theta = \pm m\lambda$
(b) $e \sin\theta = \pm \frac{(2n+1)}{2}\lambda$
(c) $e \sin\theta = \pm \left(\frac{(2m+1)}{3}\right)\lambda$
(d) none of the above

5. In single slit diffraction, it e is the width of the slit, λ is the wavelength of light and θ is the diffraction angle, then the intensity of light for the condition $e \sin\theta = (3/2)\,\lambda$ is (I_o = principal maximum) ___________.

   (a) $\frac{I_o}{4}$ (b) $\frac{I_o}{8}$ (c) $\frac{I_o}{16}$ (d) $\frac{I_o}{22}$

6. In double slit diffraction if e is the slit width and L is the separation between slits, λ is the diffracted wavelength then the condition for maxima is (m = 1, 2, 3, ...) ___________.

   (a) $(e + b)\sin\theta_n = \pm n\lambda$ (b) $e \sin\theta_n = \pm bn\lambda$ (c) $b \sin\theta_n \pm en\lambda$ (d) $(e + b)\lambda = \pm n \sin\theta_n$

7. In the diffraction of circular aperture, if f is the focal length of lens and d is the diameter of aperture and λ is the wavelength of light used, then the exact radius of Airy's disc is ___________.

   (a) $\frac{f\lambda}{d}$ (b) $\frac{fd}{\lambda}$ (c) $\frac{1.22 f\lambda}{d}$ (d) $\frac{1.22 fd}{\lambda}$

8. If light of wavelength λ is incident normal to the grating plane, the first order diffraction is observed at an angle θ, and N is the number of lines per unit length of grating, then λ is ___________.

   (a) $\frac{\sin\theta}{N}$ (b) $N \sin\theta$ (c) $\frac{N}{\sin\theta}$ (d) $\frac{1}{N\sin\theta}$

9. The capacity of an optical instrument to show separate images of very closely placed two objects is called ______.

   (a) magnifying power (b) resolving power (c) interference power (d) diffracting power

Answers

1. a 2. b 3. a 4. a 5. d 6. a 7. c 8. a 9. b

Review Questions

1. Distinguish between Fresnel and Fraunhofer class of diffraction.
2. Explain with theory the Fraunhofer diffraction at a single slit.
3. Describe Fraunhofer diffraction due to a single slit and deduce the position of maxima and minima.
4. Define dispersive power of a grating and obtain an expression for it.
5. Obtain the formula for the resolving power of an optical grating.
6. Describe the Fraunhofer diffraction at a double slit and deduce intensity distribution.
7. Give the construction and theory of a plane transmission diffraction grating and explain the formation of spectra.
7. Discuss the Fraunhofer diffraction at N-slit and obtain the intensity distribution and positions of maxima and minima.

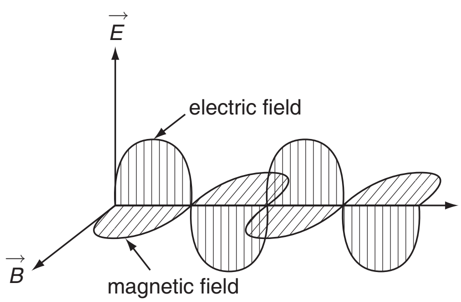
CHAPTER 3

Polarization

3.1 Introduction

The phenomenon of interference and diffraction shows that light has wave nature, but they do not confirm the longitudinal or transverse nature of light waves. The phenomenon of polarization shows the transverse nature of light rays. Light is made up of electromagnetic rays, it has electric and magnetic fields. The electric and magnetic fields are represented by electric and magnetic field vectors. These field vectors are perpendicular to the direction of propagation of light rays. The electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ are perpendicular to each other as shown in Fig. 3.1.

Figure 3.1 Light is an electromagnetic wave



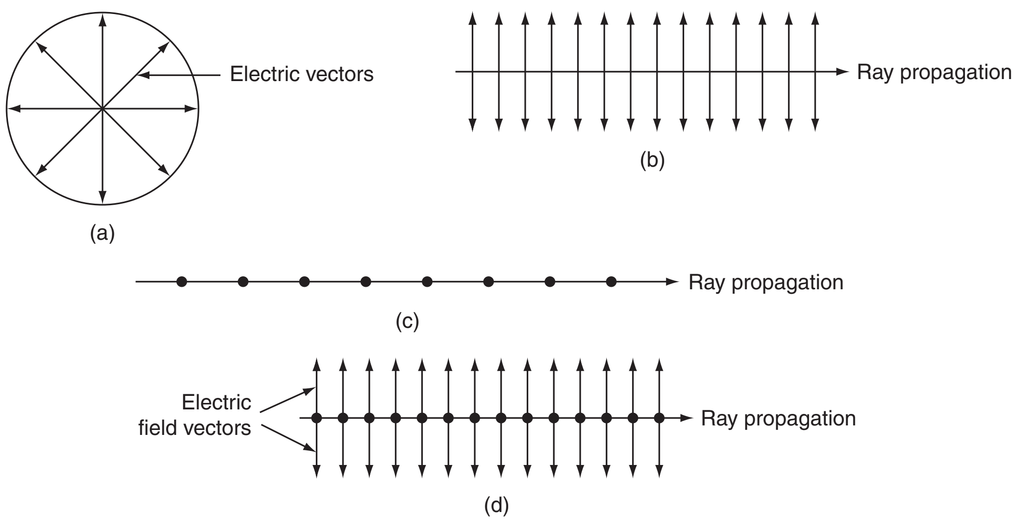
3.2 Representation of polarized and unpolarized light

Let us consider an ordinary light ray passing perpendicular to the plane of the paper and into the paper. The electric field vectors are perpendicular to the ray propagating with equal amplitude in all possible directions as shown in Fig. 3.2(a). This is the nature of unpolarized light.

The linearly polarized light is shown in Fig. 3.2(b) and in 3.2(c). In Fig. 3.2(b), the direction of electric field vectors lie in the plane of the paper and in Fig. 3.2(c) the direction of electric field vectors are perpendicular to the plane of the paper. The positions of perpendicular vectors of the ray are shown with dots.

In a light ray if the electric field vectors perpendicular to the direction of ray propagation do not have equal amplitudes, then the field vectors have been resolved along two perpendicular directions, say *X* and *Y* directions. If the resultant electric field amplitudes along *X* and *Y* directions are equal, then the ray is said to be unpolarized ray. On the other hand if they are not equal then the ray is said to be polarized along the larger resultant field amplitude. Unpolarized light is represented as a combination of that shown in Figs. 3.2(b), (c) and (d).

Figure 3.2 **(a) Electrical field vectors of unpolarized light; (b) vertically plane polarized light; (c) horizontally plane polarized light and (d) unpolarized light**



3.3 Types of polarization

We have different types of polarized light such as plane polarized, circularly polarized and elliptically polarized light.

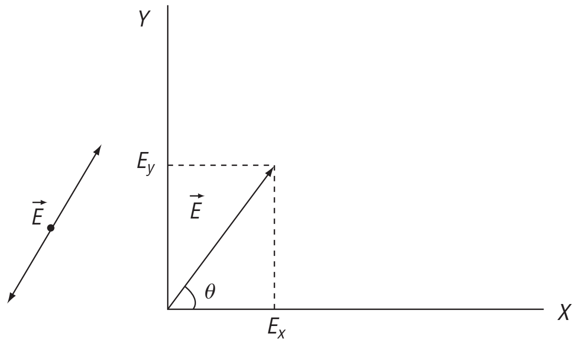
(i) Plane polarized light: The plane polarized light has been obtained from ordinary light by passing it through a tourmaline crystal. Let the electric vector $(\vec{E})$ of the plane polarized ray make an angle θ with the *X*-axis as shown in Fig. 3.3. The vector can be resolved along the *X*- and *Y*-directions.

Let the components of $(\vec{E})$ be E_x and E_y. So, the electric field vector $(\vec{E})$ may be represented as

$$\vec{E} = \vec{i}\ E_x + \vec{j}\ E_y \qquad \text{——— (3.1)}$$

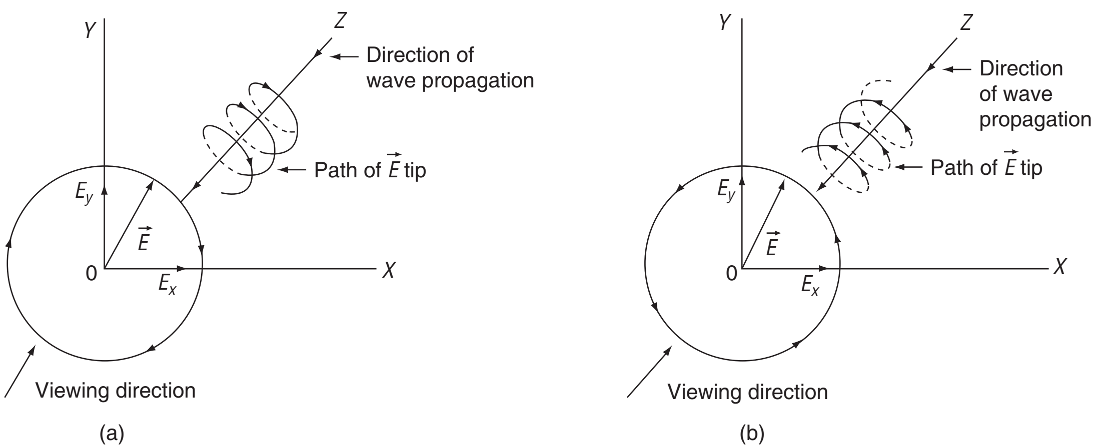
This shows that the original linearly polarized wave may be viewed as a superposition of two coherent waves having zero phase difference of which one polarized in the *Y*–*Z* plane and the other polarized in the *X*–*Z* plane.

Figure 3.3 The electric vector $\vec{E}$ of a plane polarized light



(ii) Circularly polarized light: Suppose the coherent light waves E_x and E_y are equal in magnitude and there is a phase difference of 90° (or $\pi/2$ radian) between them, then the superposition of these waves gives a circularly polarized ray. The resultant electric vector $(\vec{E})$ is constant in magnitude and rotates [as we see against the wave] about the direction of propagation such that the tip of the vector travels along a helical path. If the direction of rotation of the vector is in the clockwise direction, then it is known as right circularly polarized ray as shown in Fig. 3.4(a) or if the direction of rotation of the vector is in the anticlockwise direction, then it is known as the left circularly polarized ray as shown in Fig. 3.4(b).

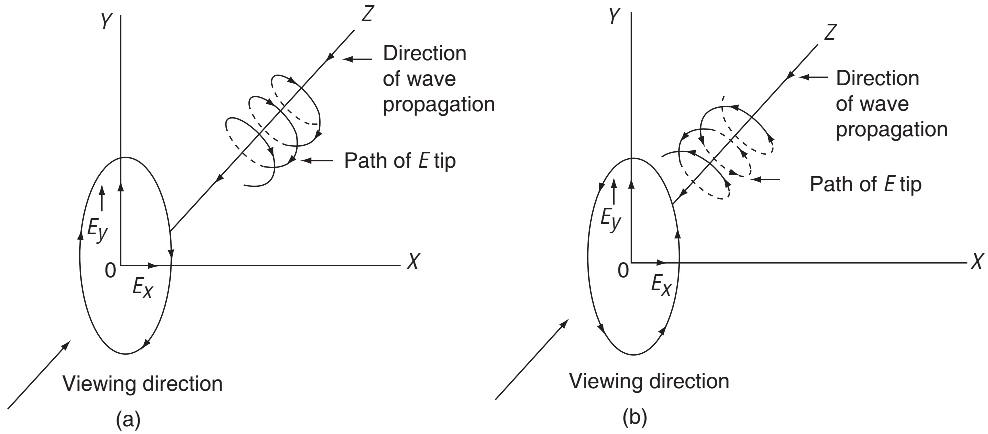
Figure 3.4 (a) Right circularly polarized ray and (b) left circularly polarized ray



(iii) Elliptically polarized light: The superposition of two perpendicularly plane polarized light waves of unequal magnitude and out of phase will produce elliptically polarized light. Let the electric vectors in the polarized light be E_x and E_y along the X-and Y-direction; then the tip of the resultant vector $(\vec{E})$ of E_x and E_y sweeps a flattened helix in space. It traces an ellipse in a plane perpendicular to the direction of ray propagation. If the rotation of the tip of the vector $(\vec{E})$ is in a clockwise direction, then it is said to be right

elliptically polarized light as shown in Fig. 3.5(a). On the other hand, if the tip of the vector $(\vec{E})$ rotates in an anticlockwise direction, then it is known as left elliptically polarized light as shown in Fig. 3.5(b).

Figure 3.5 (a) Right elliptically polarized ray and (b) left elliptically polarized ray



3.4 Polarization by reflection

In 1809, Malus, a French scientist discovered that when ordinary light is incident on the surface of a transparent medium like glass or water, then light can be partially or completely polarized on reflection. The extent of polarization of reflected light varies with the angle of incidence. In 1811, Sir David Brewster noticed the extent of variation polarization of reflected light by varying the angle of incidence on the surfaces of different transparent materials. He observed that for a particular angle of incidence [$\theta_i = \theta_p$, θ_p = angle of polarization] the reflected light is completely plane polarized as shown in Fig. 3.6. This angle of incidence is known as Brewster's angle or angle of polarization. The angle of polarization varies with material also. The Brewster's angle for glass ($\mu = 1.52$) is 57°.

Suppose the incident beam makes Brewster's angle, then the reflected light is completely plane polarized while the transmitted light is partially plane polarized is shown in Fig. 3.6. In the reflected light the vibrations of the electric vectors are perpendicular to the plane of incidence. The plane of incidence is the plane containing incident beam and the normal to the surface at the point of incidence. The intensity of reflected wave is less and it is nearly 15% intense as compared to the intensity of incident beam, while the intensity of transmitted beam is large and is partially polarized. By using a large number of thin parallel glass plates instead of single glass plate as shown in Fig. 3.7, the intensity of reflected waves is enhanced and the transmitted beam becomes more plane polarized. The vibrations of electric vectors in the transmitted beam is in the plane of incidence.

Experimentally it was found by Brewster that when the angle of incidence (θ_i) is equal to the polarizing angle (θ_p) the angle between the reflected ray and refracted ray is 90°.

Hence $\theta_p + 90° + \theta_r = 180°$ [Since angle of incidence = angle of reflection and θ_r = angle of refraction]

or $\theta_p + \theta_r = 90°$ ———— (3.2)

Figure 3.6 Polarization by reflection

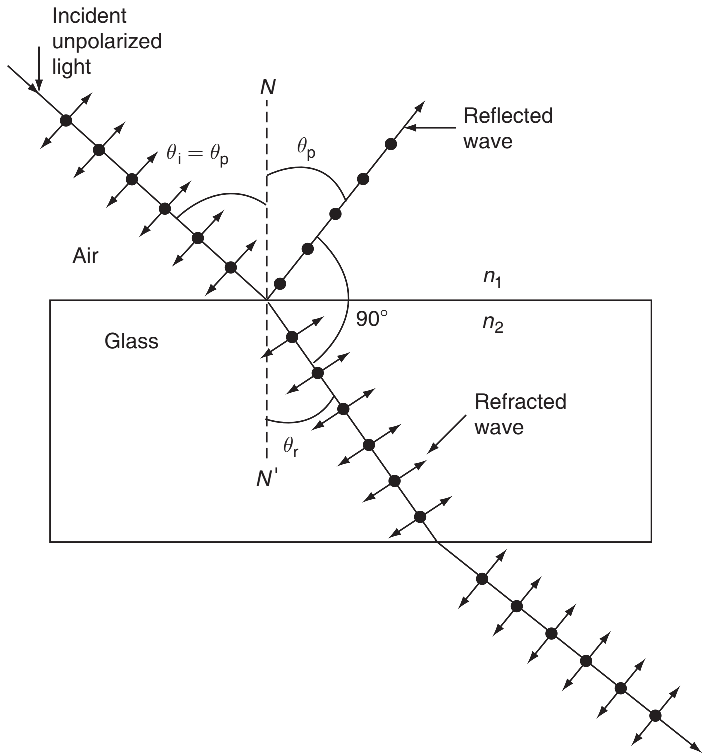
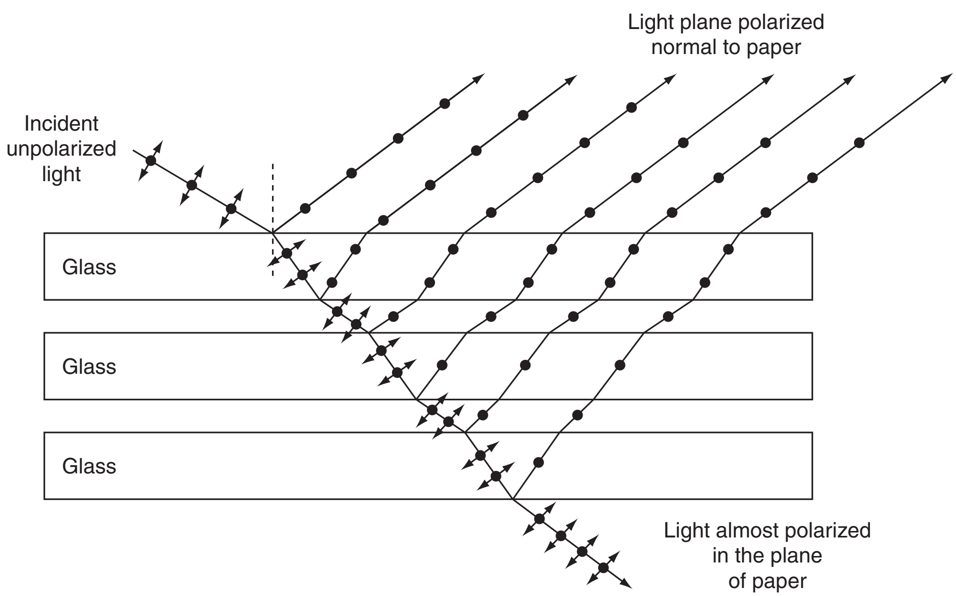


Figure 3.7 Polarization of light by a stack of glass plates



From Snell's law

$$n_1 \sin \theta_p = n_2 \sin \theta_r \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (3.3)}$$

[n_1 = refractive index of air and n_2 = refractive index of glass plate]

Using Equation (3.2) in (3.3),

$$n_1 \sin \theta_p = n_2 \sin (90^\circ - \theta_p)$$

$$= n_2 \cos \theta_p$$

$$\text{or } \tan \theta_p = \frac{n_2}{n_1} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (3.4)}$$

The above equation is known as Brewster's law, because Brewster deduced it empirically in 1812.

3.5 Malus law

Plane polarized light is obtained by passing unpolarized light through a polarizer. When plane polarized light from the polarizer is passed through the analyser, the intensity of polarized light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polarizer. This is known as Malus law.

Malus law can be proved by considering the amplitude (a) of the incident plane polarized light on the surface of the analyser as shown in Fig. 3.8. Let θ be the angle between the planes of the analyser and the polarizer.

The amplitude of incident plane polarized light parallel to the plane of transmission of the analyser is $a \cos \theta$ and perpendicular to it is $a \sin \theta$.

Only the parallel component pass through the analyser.

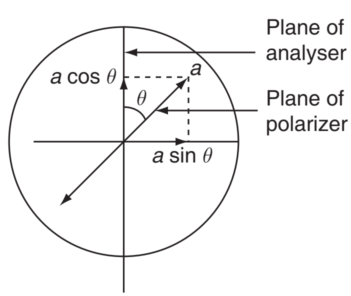
∴ The intensity of light transmitted through the analyser is

$$I = (a \cos \theta)^2 = a^2 \cos^2 \theta \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (3.5)}$$

If $I_0 = a^2$ is the intensity of plane polarized light on the surface of the analyser then Equation (3.5) becomes

$$I = I_0 \cos^2\theta \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (3.6)}$$

Figure 3.8 Malus law



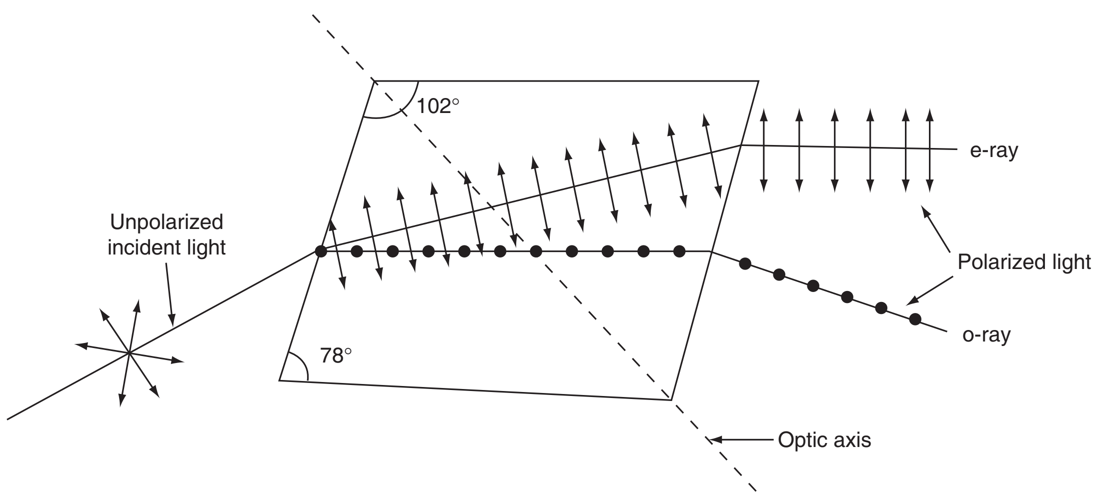
When $\theta = 0, I = I_0$ and if $\theta = 90°$ then $I = 0$. The above results have been proved for tourmaline crystals, Nicol prisms, etc.

3.6 Double refraction

When a beam of unpolarized light passes through anisotropic crystals such as quartz or calcite, the beam will split up into two refracted beams. This is known as double refraction or birefringence. The direction in which the ray of transmitted light does not suffer double refraction inside the crystal is known as the optic axis. If only one optic axis is present in a crystal then it is called uni-axial crystal. On the other hand if two optic axes are present in a crystal then it is known as biaxial crystals.

Double refraction in calcite crystal is shown in Fig. 3.9. The refracted beams are plane polarized. One beam is polarized along the direction of the optic axis and is known as extraordinary ray (e-ray), while the other refracted beam is polarized along the direction perpendicular to the optic axis and is known as ordinary ray (o-ray).

Figure 3.9 Double refraction in calcite crystal

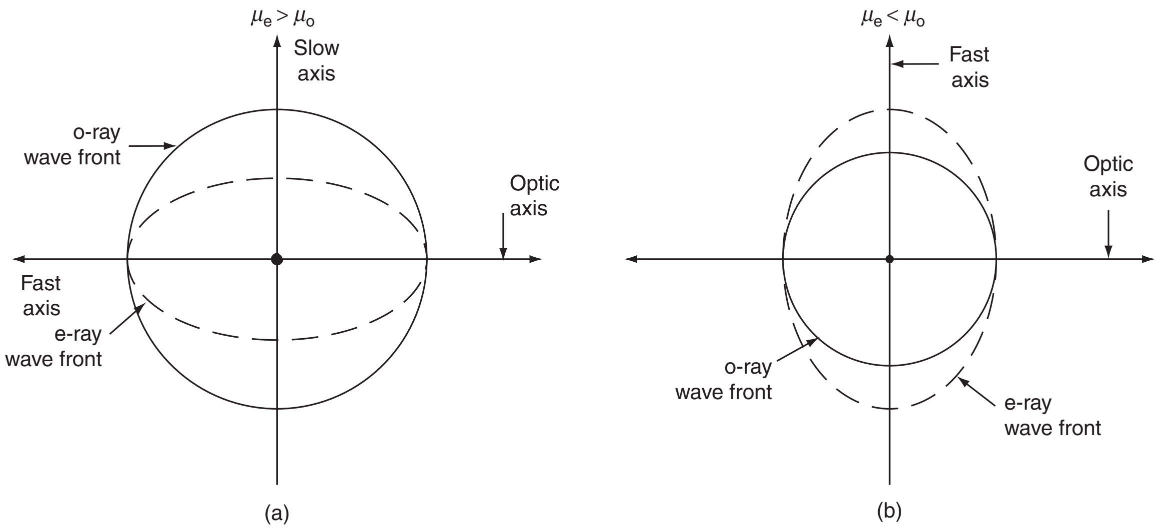


The wave fronts of e-ray and o-ray are shown for quartz and calcite crystals in Fig. 3.10.

The velocity of o-ray is the same in all directions of a crystal, whereas the extraordinary ray travels with different velocities in different directions. The velocities of o-ray and e-ray are the same along the optic axis. The velocity of e-ray is less than that of o-ray in quartz crystal, this is a positive crystal. The velocity of e-ray is more than that of o-ray in calcite crystal, this is a negative crystal.

Along the optic axis, the refractive index $\mu_e = \mu_0$. So the wave fronts of e-ray and o-ray will coincide. In quartz crystal $\mu_e > \mu_o$ in all other directions, and it is very large in the direction perpendicular to the optic axis. Calcite crystallizes in rhombohedral (trigonal) crystal system. The diagonal line that passes through the blunt corners is the optic axis. In calcite $\mu_e < \mu_o$ in all directions except the optic axis. So, the speed of e-ray is larger than that of o-ray. μ_e is very much less in the direction perpendicular to the optic axis.

Figure 3.10 (a) Wave fronts in quartz crystal and (b) wave fronts in calcite crystal



3.7 Nicol prism

Nicol prism is an optical device used to produce and analyse plane polarized light. This was invented by William Nicol in the year 1828. Nicol prism is made from a double refracting calcite crystal. As shown in Fig. 3.11, a calcite crystal whose length is three times its breath is taken. The corners A′, G′ are blunt and A′C G′E is the principal section with $\angle A'CG' = 71°$. The end faces A′BCD and EFG′H are grounded so that the angle ACG = 68°. The crystal is cut along the plane AKGL. The cut surfaces are polished until they are optically flat and cemented together with Canada balsam.

The refractive index of Canada balsam (1.55) is in between the refractive indices of ordinary (1.658) and extraordinary (1.486) rays in calcite crystal.

The section ACGE of Fig. 3.11 is shown separately in Fig. 3.12. In Fig. 3.12, the diagonal AG represents the Canada balsam layer. A beam of unpolarized light is incident parallel to the lower edge on the face ABCD. They are doubly refracted on entering into the crystal. From the refractive index values, we know that the Canada balsam acts as a rarer medium for the ordinary ray and it acts as a denser medium for extraordinary ray.

When the angle of incidence for ordinary ray on the Canada balsam is greater than the critical angle then total internal reflection takes place, while the extraordinary ray gets transmitted through the prism.

Nicol prism can be used as an analyzer. This is shown in Fig. 3.13.

Two Nicol prisms are placed adjacently as shown in Fig. 3.13(a). One prism acts as a polarizer and the other act as an analyzer. The extraordinary ray passes through both the prisms. If the second prism is slowly rotated, then the intensity of the extraordinary ray decreases. When they are crossed, no light come out of the second prism because the e-ray that comes out from first prism will enter into the second prism and act as an ordinary ray. So, this light is reflected in the second prism. The first prism is the polarizer and the second one is the analyser.

Figure 3.11 Calcite crystal

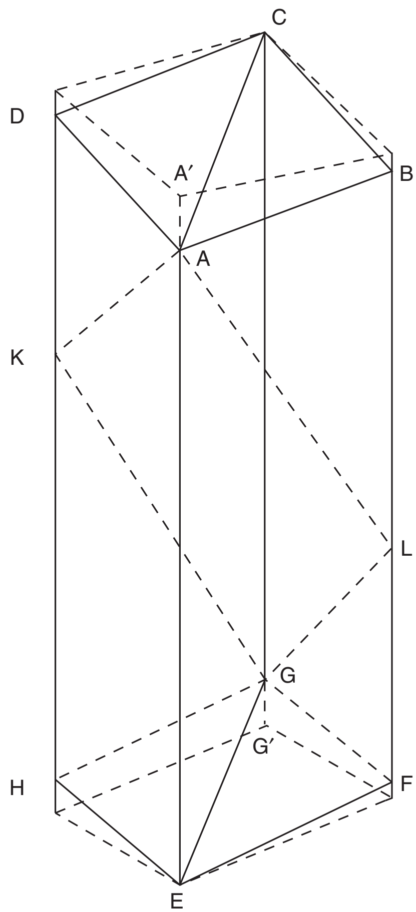


Figure 3.12 Production of plane polarized light using Nicol prism

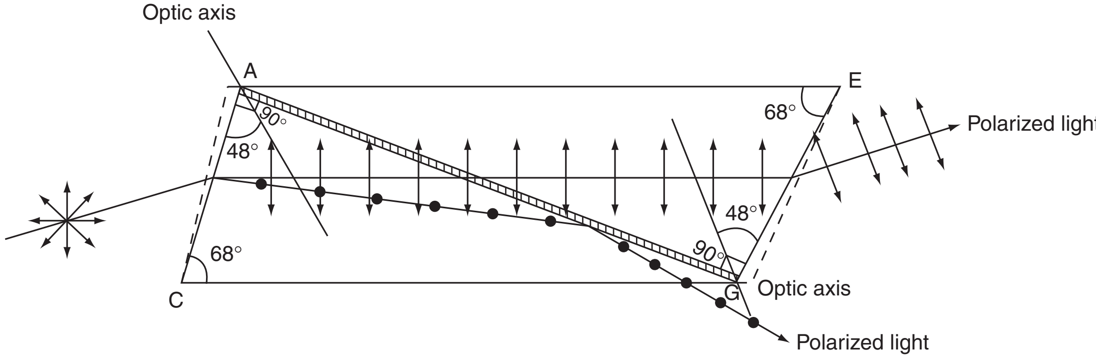
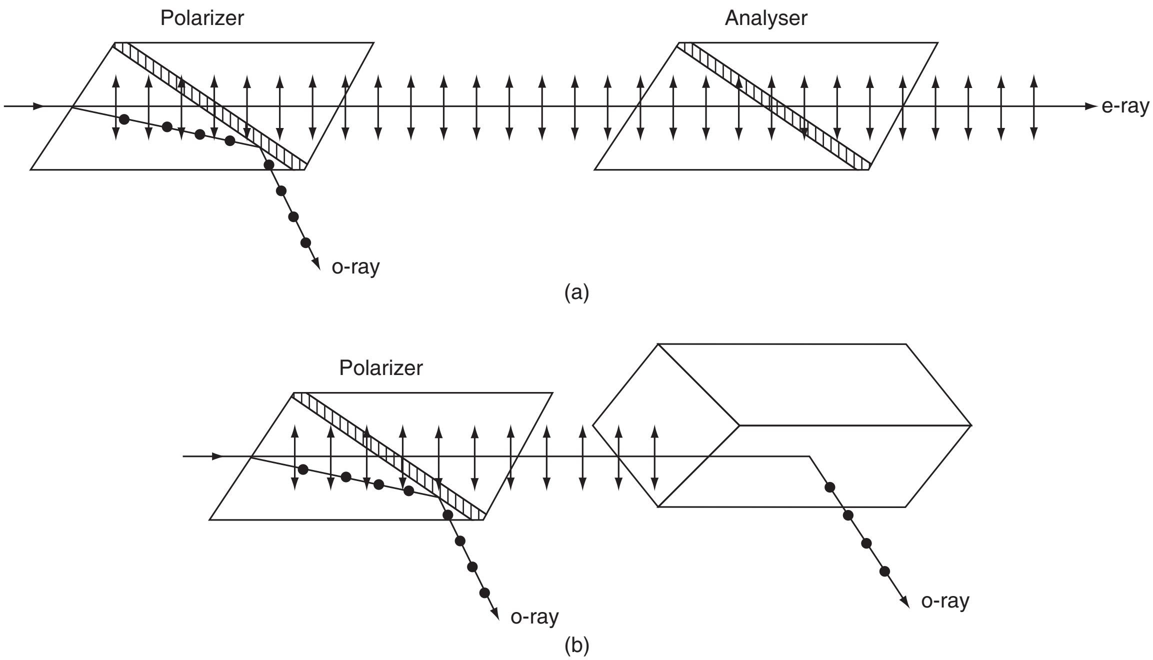


Fig. 3.13 (a) Parallel Nicols and (b) crossed Nicols



3.8 Quarter-wave plate

A quarter-wave $(\lambda/4)$ plate is a thin double refracting crystal of calcite or quartz, cut and polished parallel to its optic axis to a thickness 'd' such that it produces a path difference of $\lambda/4$ or phase difference of $\pi/2$ between the o-ray and e-ray when plane polarized light incident normally on the surface and passes through the plate.

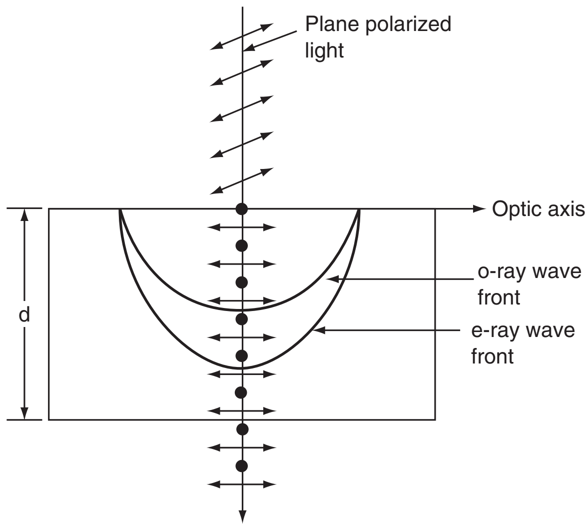
As shown in Fig. 3.14, consider a calcite crystal of thickness '*d*'. The optic axis is parallel to the surface. When a plane polarized light is incident normally on the surface, then the light will split up into o-ray and e-ray. These rays travel with different speeds in the crystal. In calcite crystal the e-ray travel faster than o-ray. Hence the refractive index of o-ray (μ_o) is higher than the refractive index of e-ray (μ_e) in the crystal. The optical path covered by the o-ray as it pass through the crystal of thickness '*d*' is $\mu_o d$. Similarly the optical path covered by the e-ray as it pass through the crystal of thickness '*d*' is $\mu_e d$.

$$\therefore \text{The path difference, } \Delta = \mu_o d - \mu_e d = d(\mu_o - \mu_e)$$

As the crystal is a quarter-wave plate, it introduces a path difference of $\lambda/4$ between o-ray and e-ray. Therefore $\Delta = \dfrac{\lambda}{4}$.

So we can write, $\lambda/4 = d(\mu_o - \mu_e)$ or $d = \lambda/4(\mu_o - \mu_e)$

Figure 3.14 Propagation of polarized light in calcite quarter-wave plate



For some other crystals like quartz, where $\mu_e > \mu_o$, the thickness of the quarter wave plate $d = \lambda / 4(\mu_e - \mu_o)$. Using the above equation, the thickness of quarter wave plate can be estimated.

3.9 Half-wave plate

Similar to the quarter-wave plate, a half-wave plate introduces a path difference of $\lambda / 2$ or phase difference of π between o-ray and e-ray. Let 'd' be the thickness of the half-wave plate.

For a half-wave plate, the path difference, $\Delta = \lambda / 2 = d(\mu_o - \mu_e)$ or $d = \lambda / 2(\mu_o - \mu_e)$ in calcite crystal half-wave plate. In the case of quartz, $d = \lambda / 2(\mu_e - \mu_o)$. Using the above equation, the thickness of half-wave plate can be estimated.

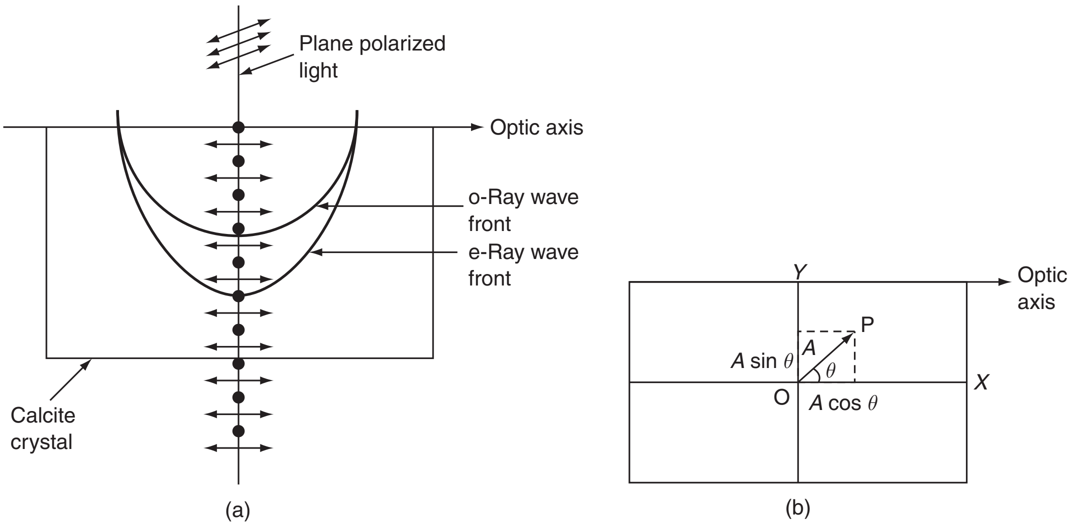
3.10 Theory of circular and elliptically polarized light

A beam of plane polarized light can be obtained from a Nicol prism. This beam of plane polarized light is made incident normally on the surface of a calcite crystal cut parallel to its optic axis.

As shown in Fig. 3.15(a), let the plane of polarization of the incident beam make an angle θ with the optic axis and let the amplitude of this incident light be A.

As polarized light enters into the calcite crystal, it will split into two components, e-ray and o-ray. The e-ray amplitude $A \cos \theta$ is parallel to the optic axis and o-ray amplitude $A \sin \theta$ is perpendicular to the optic axis. As shown in Fig. 3.15(a), inside the crystal e-ray and o-ray travel in the same direction with different amplitudes. On emerging from the crystal the rays have a phase difference 'δ' (say), depending on the thickness 'd' of the crystal. Let ν be the frequency of light. The e-ray and

Figure 3.15 (a) Plane wave incident on calcite crystal and (b) e-ray and o-ray light amplitudes in calcite crystal



o-ray can be represented in terms of simple harmonic motions, at right angles to each other having a phase difference 'δ'. The e-ray moves faster than o-ray in calcite crystal. Hence, the instantaneous displacements are

$x = A \cos \theta \sin (\omega t + \delta)$ for e-ray ___________ (3.7)

and $y = A \sin \theta \sin \omega t$ for o-ray ___________ (3.8)

where $\omega = 2\pi\nu$

Let $A \cos \theta =$ a and $A \sin \theta = b$, then Equations (3.7) and (3.8) become

$x = a \sin (\omega t + \delta)$ ___________ (3.9)

$y = b \sin \omega t$ ___________ (3.10)

From Equation (3.10)

$$\frac{y}{b} = \sin \omega t \quad \text{and} \quad \cos \omega t = \sqrt{1 - \frac{y^2}{b^2}} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_ (3.11)}$$

From Equation (3.9)

$$\frac{x}{a} = \sin \omega t \cos \delta + \cos \omega t \sin \delta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_ (3.12)}$$

Substituting Equation (3.11) in (3.12),

$$\frac{x}{a} = \frac{y}{b} \cos \delta + \sqrt{1 - \frac{y^2}{b^2}} \sin \delta$$

$$\text{or} \quad \frac{x}{a} - \frac{y}{b} \cos \delta = \sqrt{1 - \frac{y^2}{b^2}} \sin \delta$$

Squaring both sides

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\cos^2\delta-2\frac{x}{a}\frac{y}{b}\cos\delta=\left(1-\frac{y^2}{b^2}\right)\sin^2\delta$$

or $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}\cos^2\delta+\dfrac{y^2}{b^2}\sin^2\delta-2\dfrac{x}{a}\dfrac{y}{b}\cos\delta=\sin^2\delta$

or $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{2xy}{ab}\cos\delta=\sin^2\delta$ ____________ (3.13)

This is the general equation for an ellipse.
Special cases: (1) Suppose the phase difference $\delta = 0$.
Then $\sin\delta = 0$ and $\cos\delta = 1$
Equation (3.13) becomes

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}=0$$

or $\left(\dfrac{x}{a}-\dfrac{y}{b}\right)^2=0$

or $\dfrac{x}{a}-\dfrac{y}{b}=0$

or $y=\dfrac{b}{a}x$ ____________ (3.14)

This is the equation for a straight line. So the light that comes out of the crystal is plane polarized.

Case (2): Suppose $\delta=\dfrac{\pi}{2},\ \dfrac{3\pi}{2},\ \dfrac{5\pi}{2}\cdots\left(\dfrac{2n+1}{2}\right)\pi$

$n = 0, 1, 2, 3, \ldots$
Then, Equation (3.13) becomes

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (3.15)}$$

Equation (3.15) represents an ellipse. So, the emergent light from the crystal will be elliptically polarized.

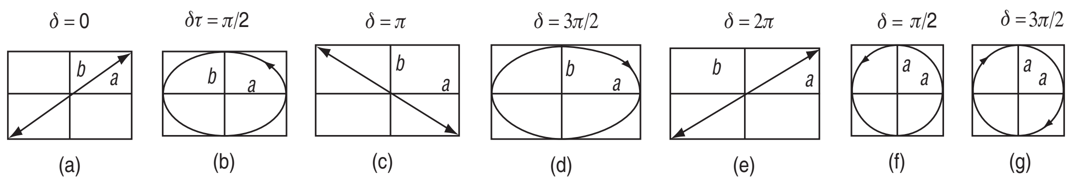
Case (3): Suppose $\delta=\dfrac{\pi}{2}$ and $a=b$

Then, from Equation (3.15)

$$x^2+y^2=a^2 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (3.16)}$$

Equation (3.16) represents a circle. So, the emergent light from the crystal will be circularly polarized. Circularly polarized light can also be produced when the incident plane polarized light makes an angle of 45° with the optic axis. The linear, elliptical and circular polarizations for different values of δ are shown in Fig. 3.16.

Figure 3.16 The different polarizations for different δ values



Formulae

1. Brewster's law, $\tan \theta_p = \mu$ also $\theta_p + \theta_r = 90°$
2. Malus law, $I = I_o \cos^2 \theta$
3. Quarter-wave plate,
$$d = \frac{\lambda}{4(\mu_o - \mu_e)} \text{ for calcite crystal}$$
$$d = \frac{\lambda}{4(\mu_e - \mu_o)} \text{ for quartz crystal}$$
4. Half-wave plate,
$$d = \frac{\lambda}{2(\mu_e - \mu_o)} \text{ for calcite crystal}$$
$$d = \frac{\lambda}{2(\mu_o - \mu_e)} \text{ for quartz crystal}$$

Solved Problems

1. *Calculate the Brewster angle for (i) ethylalcohol for which μ = 1.361 and (ii) carbontetrachloride for which μ = 1.461.*

Sol: Brewster's law is $\tan \theta_P = \mu$

For ethylalcohol, $\mu = 1.361$

so $\theta_p = \tan^{-1} 1.361 = 53° \, 41' \, 36''$

For carbon tetrachloride, $\mu = 1.461$

so $\theta_p = \tan^{-1} 1.461 = 55° \, 36' \, 35''$

2. *For flint glass material the Brewster angle is $(\theta_p) = 62° \, 24'$. Find the refractive index of the material.*

Sol: Brewster angle for flint glass, $\theta_p = 62° \, 24'$

Refractive index, μ = ?

Brewster's law is $\tan \theta_p = \mu$

so $\mu = \tan 62° \, 24' = 1.9128$

3. *The refractive index of a polarizer is 1.54. Find the polarization angle and angle of refraction.*

Sol: The refractive index of a polarizer, $\mu = 1.54$

Brewster's law is $\tan \theta_p = \mu$

The polarization angle, $\theta_p = \tan^{-1} \mu = \tan^{-1} 1.54 = 57°$

we know

$$\theta_p + \theta_r = 90°$$
$$\theta_r = 90° - 57° = 33°$$

4. *The refractive indices of mica for ordinary and extraordinary rays are 1.586 and 1.592 with a wavelength of 5460 Å. Find the thickness of mica sheet to act as a quarter wave plate.*

Sol: Wavelength of light, λ = 5460 Å = 5460×10^{-10} m
refractive index of o-ray, $\mu_o = 1.586$
refractive index of e-ray, $\mu_e = 1.592$

For quarter wave plate, $d = \dfrac{\lambda}{4(\mu_e - \mu_o)}$

$$= \frac{5460 \times 10^{-10}}{4(1.592 - 1.586)} = \frac{0.546 \times 10^{-6}}{0.024} \text{ m} = 22.75 \times 10^{-6} \text{m}$$

5. *For calcite crystal μ_e = 1.486 and μ_o = 1.658 for a light of wavelength λ = 5893 × 10^{-10}m. Find the thickness of the calcite crystal to produce circularly polarized light.*

Sol: To produce circularly polarized light, the path difference between the two rays should be $\lambda/4$. So quarter wave plate is to be required. So

$$d = \frac{\lambda}{4(\mu_o - \mu_e)}$$

Wavelength of light, λ = 5893 Å = 5893×10^{-10} m

$\mu_e = 1.486$ and $\mu_o = 1.658$

Hence, $d = \dfrac{5893 \times 10^{-10} m}{4(1.658 - 1.486)} = 8.5654 \times 10^{-7}$ m

Multiple-choice Questions

1. Brewster's angle for glass is ___________.
   (a) 57° (b) 45° (c) 60° (d) 75°
2. Along the optical axis ___________.
   (a) velocity of o-rays is larger than e-ray
   (b) velocity of o-ray is less then e-ray
   (c) both the rays have the same velocity
   (d) none of the above
3. $\mu_e > \mu_o$ for ___________.
   (a) quartz (b) calcite (c) both a and b (d) none of the above
4. $\mu_o > \mu_e$ for ___________.
   (a) quartz (b) calcite (c) both a and b (d) none of the above
5. Nicol prism is used to ___________.
   (a) produce polarized light
   (b) analyse polarized light
   (c) both a and b
   (d) none of the above
6. The refractive index of Canada balsam is ___________.
   (a) less than o-ray of calcite
   (b) larger than e-ray of calcite
   (c) in between o-ray and e-ray of calcite
   (d) all the above

7. Quarter-wave plate produces a path difference of ______ between o-ray and e-ray.

   (a) $\frac{\lambda}{2}$ (b) $\frac{\lambda}{4}$ (c) $\frac{\lambda}{6}$ (d) $\frac{\lambda}{8}$

8. Half-wave plate produces a path difference of _____ between o-ray and e-ray.

   (a) $\frac{\lambda}{2}$ (b) $\frac{\lambda}{4}$ (c) $\frac{\lambda}{6}$ (d) $\frac{\lambda}{8}$

9. If the electric field vectors are present in different directions in a plane perpendicular to ray directions, then it is said to be ___________.

   (a) plane polarized ray (b) elliptically polarized ray
   (c) circularly polarized ray (d) unpolarized ray

10. If the electric vectors of a light ray are in a single plane along the ray direction, then it is said to be ___________.

   (a) plane polarized ray (b) elliptically polarized ray
   (c) circularly polarized ray (d) unpolarized ray

11. Plane of polarization means ___________.

   (a) a plane normal to the plane of vibration (b) a plane normal to the plane of vibration
   (c) both a and b (d) none of the above

12. To produce circularly polarized light, the phase difference between two perpendicularly polarized light rays of equal magnitude should be ___________.

   (a) 45° (b) 90° (c)180° (d) 270°

13. Elliptically polarized light is produced by the superposition of ___________.

   (a) two parallel plane polarized rays
   (b) two perpendicular plane polarized rays
   (c) the magnitude of rays is unequal and out of phase
   (d) both b and c

14. Polaroid produces plane polarized light by ___________.

   (a) double refraction (b) selective absorption of light
   (c) both a and b (d) none of the above

15. Polaroids are ___________.

   (a) used to produce and analyse polarized light
   (b) used to vary intensity of light
   (c) used to suppress headlight glare in motor vehicles
   (d) all the above

Answers

1. a 2. c 3. a 4. b 5. c 6. d 7. b 8. a 9. d 10. a 11. a
12. b 13. d 14. c 15. d

Review Questions

1. Describe the method of producing plane polarized light by refraction. Explain Brewster's law.
2. Write short notes on the law of Malus.

3. Explain the construction and working of Nicol prism.
4. Explain the phenomenon of double refraction.
5. Explain quarter wave plate and half wave plate.
6. What is polarization of light? Describe various methods of producing plane polarized light.
7. Write an essay on the linearly, circularly and elliptically polarized light.

CHAPTER 4

Crystal Structures

4.1 Introduction

Matter exists in three different states; they are gaseous, liquid and solid states. In gaseous and liquid states, the atoms or molecules of the substance move from one place to other, and there is no fixed position of atoms in the substance. In solids, the positions of the atoms or molecules are fixed and may or may not be present periodically at regular intervals of distance. If the atoms or molecules in a solid are periodical at regular intervals of distances in three-dimensional space, then that solid is known as crystalline solid. If the atoms or molecules do not have such a periodicity in a solid, then that solid is known as amorphous solid. When the periodicity of atoms or molecules is extended throughout the solid, then the solid is known as single crystalline solid. If the periodicity of atoms or molecules is extended up to small regions called grains and if these grains are very large in number, and are of different sizes in the solid, such a material is known as polycrystalline solid. The study of geometric form and other physical properties of crystalline solids by using X-rays, electron beams and neutron beams constitute the science of crystallography.

Distinction between crystalline and amorphous solids

Crystalline Solids	Amorphous Solids
1. The atoms or molecules of the crystalline solids are periodic in space.	1. The atoms or molecules of the amorphous solids are not periodic in space.
2. Some crystalline solids are anisotropic i.e., the magnitude of physical properties [such as refractive index, electrical conductivity, thermal conductivity, etc.,] are different along different directions of the crystal.	2. Amorphous solids are isotropic, i.e. the magnitude of the physical properties are same along all directions of the solid.
3. Crystalline solids have sharp melting points.	3. Amorphous solids do not posses sharp melting points.

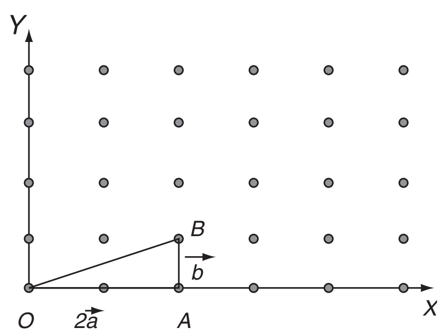
4. Breaks are observed in the cooling curve of a crystalline solid.	4. Breaks are not observed in the cooling curve.
5. A crystal breaks along certain crystallographic planes.	5. When an amorphous solid breaks, the broken surface is irregular because it has no crystal planes.

4.2 Space lattice (or) crystal lattice

In a solid crystalline material, the atoms or molecules are arranged regularly and periodically in three dimensions. To explain crystal symmetries easily, it is convenient to represent an atom or a group of atoms that repeats in three dimensions in the crystal as a unit. If each such unit of atoms or atom in a crystal is replaced by a point in space, then the resultant points in space are called space lattice. Each point in space is called a lattice point and each unit of atoms or atom is called basis or pattern. A space lattice represents the geometrical pattern of crystal in which the surroundings of each lattice point is the same.

If the surroundings of each lattice point is same or if the atom or all the atoms at lattice points are identical, then such a lattice is called Bravais lattice. On the other hand, if the atom or the atoms at lattice points are not same, then it is said to be a non-Bravais lattice. Figure 4.1 shows a two-dimensional lattice.

Figure 4.1 Two-dimensional lattice

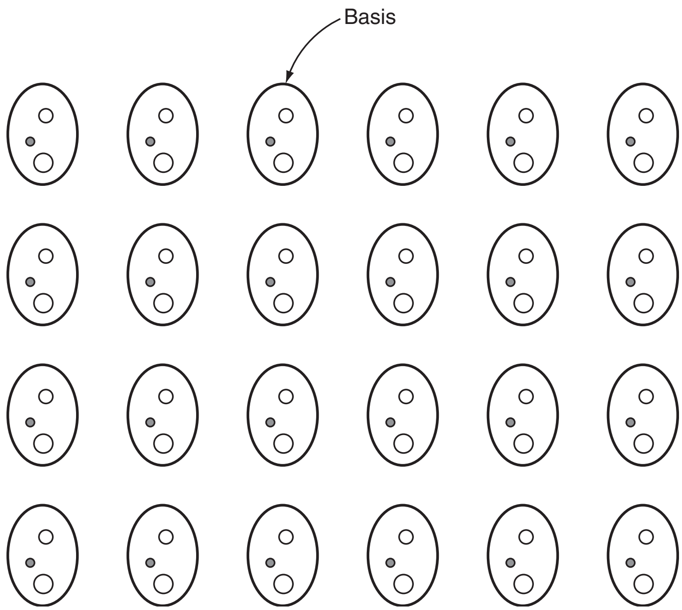


To represent translational vectors or basis vectors, consider a co-ordinate system with its origin at the lattice point '*O*'. Let $\overrightarrow{OA} = 2\vec{a}$ and $\overrightarrow{AB} = \vec{b}$, such that $\overrightarrow{OB} = 2\vec{a} + \vec{b}$. where $\vec{a}$ and $\vec{b}$ are called translational or basis vectors along *X* and *Y* directions. The position vector $\vec{R}$ of any lattice point can be represented as $\vec{R} = n_1\vec{a} + n_2\vec{b}$. where n_1 and n_2 are integers, their values depend on the position of the lattice point under consideration with respect to the origin. In three dimensions, the position vector of a point can be expressed as $\vec{R} = n_1\vec{a} + n_2\vec{b} + n_3\vec{c}$, where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the translational or basis vectors along *X*, *Y* and *Z* directions, respectively. They are also called translational primitives.

4.3 The basis and crystal structure

The crystal structure is formed by associating every lattice point with an assembly of atoms or molecules or ions, which are identical in composition, arrangement and orientation, is called as the basis. The atomic arrangement in a crystal is called crystal structure. If the basis is substituted for the lattice points, then the resulting structure is called crystal structure as shown in Fig. 4.2. Thus lattice + basis = crystal structure. The basis shown in Fig. 4.2 contains three different atoms. In copper and sodium crystals the basis is single atoms; in NaCl, the basis is diatomic and in CaF_2 the basis is triatomic.

Figure 4.2 Two-dimensional crystal structure

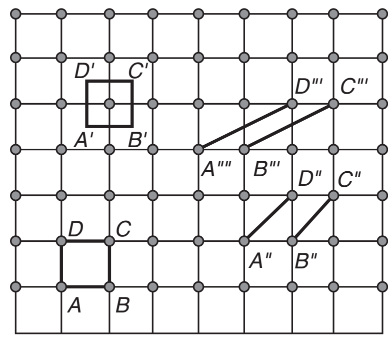


4.4 Unit cell and lattice parameters

Unit cells for most of the crystals are parallelopipeds or cubes having three sets of parallel faces. A unit cell is the basic structural unit or building block of the crystal. A unit cell is defined as the smallest parallelopiped volume in the crystal, which on repetition along the crystallographic axes gives the actual crystal structure or the smallest geometric figure, which on repetition in three-dimensional space, gives the actual crystal structure called a unit cell. The choice of a unit cell is not unique but it can be constructed in a number of ways; Fig. 4.3 shows different ways of representing unit cells in a two-dimensional lattice. A unit cell can be represented as $ABCD$ or $A'B'C'D'$ or $A''B''C''D''$, etc.

To define the lattice parameters, first we define crystallographic axes. These axes are obtained by the intersection of the three non-coplanar faces of the unit cell. The angle between these faces or crystallographic

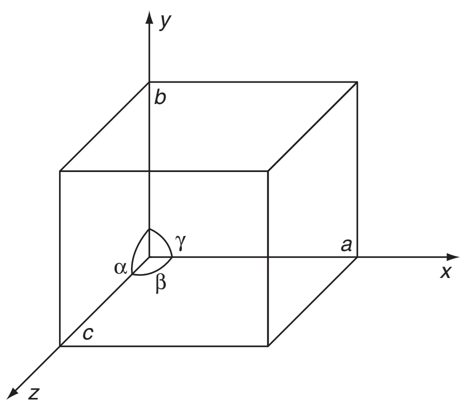
Figure 4.3 Unit cells in crystal lattice



axes are known as interfacial or interaxial angles. The angles between the axes Y and Z is $\alpha,$ between Z and X is β and between X and Y is γ. The translational vectors or primitives a, b, c of a unit cell along X, Y, Z axes and interaxial angles α, β, γ are called cell parameters. These cell parameters are shown in Fig. 4.4.

The cell parameters determine the actual size and shape of the unit cell. The unit cell formed by primitives is called a primitive unit cell. A primitive unit cell contains only one lattice point. If a unit cell contains more than one lattice point, then it is called non-primitive or multiple cells. For example, BCC and FCC are non-primitive unit cells.

Figure 4.4 Unit cell parameters



4.5 Crystal systems and Bravais lattices

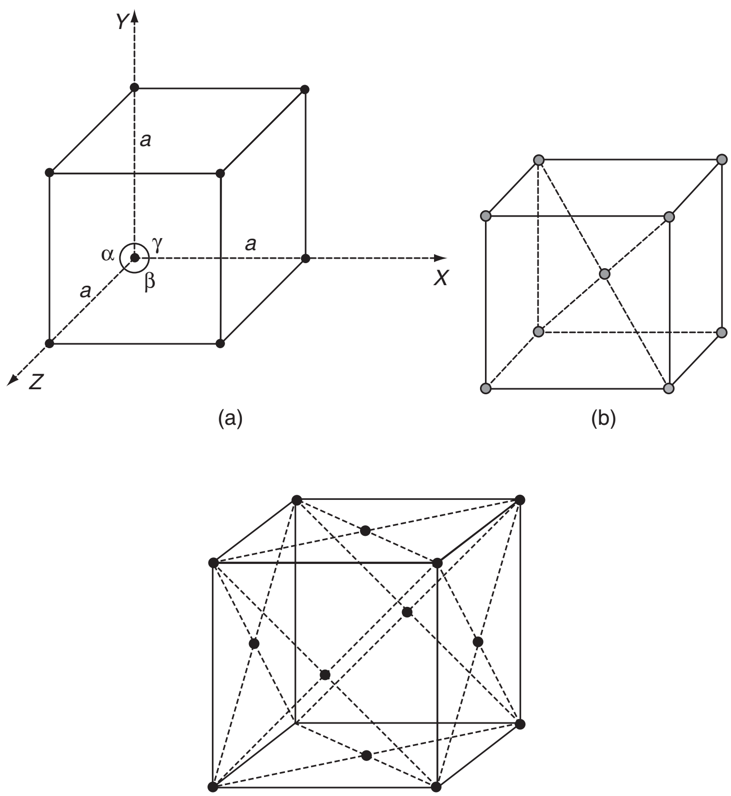
For representing the type of distribution of lattice points in space, seven different co-ordinate systems are required. These co-ordinate systems are called crystal systems. The crystal systems are named on the basis of geometrical shape and symmetry. The seven crystal systems are: (1) Cubic (2) Tetragonal (3) Orthorhombic (4) Monoclinic (5) Triclinic (6) Rhombohedral (or Trigonal) and (7) Hexagonal. Space lattices are classified according to their symmetry. In 1948, Bravais showed that 14 lattices are sufficient to describe all crystals. These 14 lattices are known as Bravais lattices and are classified into 7 crystal systems based on cell parameters.

The Bravais lattices are categorized as primitive lattice (P); body-centred lattice (I); face-centred lattice (F) and base-centred lattice (C). These seven crystal systems and Bravais lattices are described below.

1. Cubic crystal system: In this crystal system, all the unit cell edge lengths are equal and are at right angles to one another i.e., $a = b = c$ and $\alpha = \beta = \gamma = 90°$. In cubic system, there are three Bravais lattices; they are simple (primitive); body-centred and face-centred. Examples for cubic system are Au, Cu, Ag, NaCl, diamond, etc.

In simple cubic lattice, lattice points or atoms are present at the corners of the cube. In body-centred cube, atoms are present at the corners and one atom is completely present at the centre of the cube. In the case of face-centred cube, atoms are present at corners and at the centres of all faces of cube.

Figure 4.5 Cubic crystal system: (a) simple cubic (P); (b) body-centred cube (I) and (c) face-centred cube (F)



(c)

2. Tetragonal crystal system: In this crystal system, two lengths of the unit cell edges are equal whereas the third length is different. The three edges are perpendicular to one another i.e., $a = b \neq c$ and $\alpha = \beta = \gamma = 90°$. In tetragonal system, there are two Bravais lattices; they are simple and body-centred. These are shown in Fig. 4.6. Examples for tetragonal crystal systems are TiO_2, SnO_2, etc.

3. Orthorhombic crystal system: In this crystal system, unit cell edge lengths are different and they are perpendicular to one another i.e., $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90°$. There are four Bravais lattices in this

Figure 4.6 Tetragonal crystal system

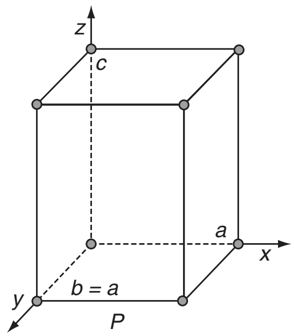
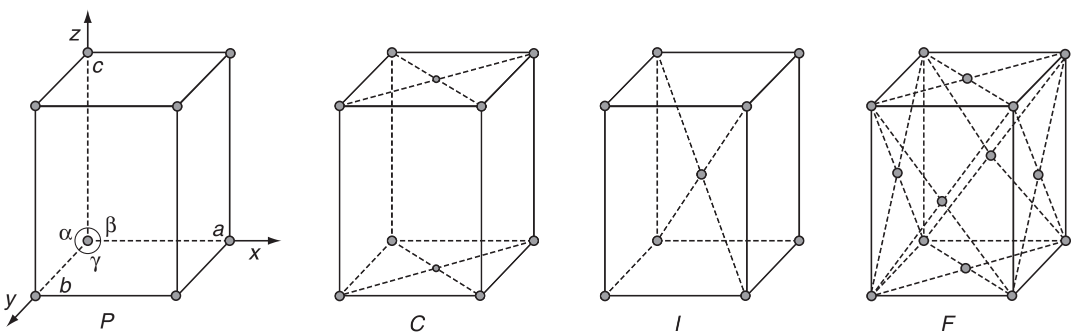


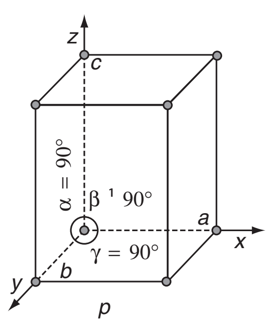
Figure 4.7 Orthorhombic crystal system



system. They are simple, face centred, body centred and base centred. These are shown in Fig. 4.7. Examples for orthorhombic crystal system are $BaSO_4$, K_2SO_4, $SnSO_4$, etc.

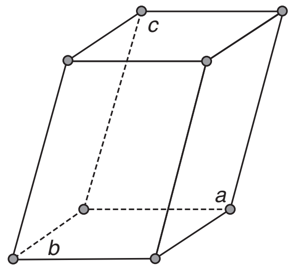
4. Monoclinic crystal system: In this crystal system, the unit cell edge lengths are different. Two unit cell edges are not perpendicular, but they are perpendicular to the third edge i.e., $a \neq b \neq c$; $\alpha = \gamma = 90^\circ \neq \beta$. This crystal system has two Bravais lattices; they are simple and base centred. These are shown in Fig. 4.8. Examples for Monoclinic crystal system are $CaSO_4.2H_2O$ (gypsum), Na_3AlF_6 (cryolite), etc.

Figure 4.8 Monoclinic crystal system



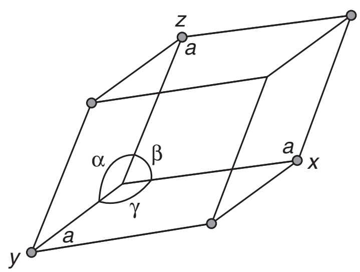
5. Triclinic crystal system: In this crystal system, the unit cell edge lengths are different and are not perpendicular i.e., $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90°$ and all the angles are different. This crystal exists in primitive cell only. This is shown in Fig. 4.9. Examples for triclinic crystal system are $K_2Cr_2O_7$, $CuSO_4 . 5H_2O$, etc.

Figure 4.9 **Triclinic crystal system**



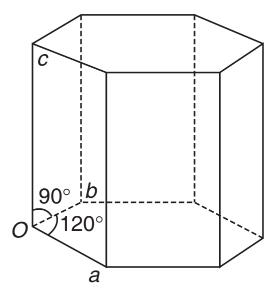
6. Rhombohedral [Trigonal] crystal system: In this crystal system, all the lengths of unit cell edges are equal. The angles between the axes are equal but other than 90°, i.e. $a = b = c$ and $\alpha = \beta = \gamma \neq 90°$. The Bravais lattice is simple only as shown in Fig. 4.10. Examples for Rhombohedral crystal system are As, Bi, Sb, etc.

Figure 4.10 **Rhombohedral crystal system**



7. Hexagonal crystal system: In this crystal system, two sides of the unit cell edge lengths are equal and the angle between these edges is 120°. These two edges are perpendicular to the third edge, and not equal in length i.e., $a = b \neq c$ and $\alpha = \beta = 90°$; $\gamma = 120°$. The Bravais lattice is primitive only. This is shown in Fig. 4.11.

Figure 4.11 **Hexagonal crystal system**



The 14 Bravais lattices of 7 crystal systems are shown in the table below.

Sl. No	Crystal System	Types of Bravais Lattices	No. of Bravais Lattices	Relation between Lengths and Angles
1	Cubic	P, I, F	3	$a = b = c$ $\alpha = \beta = \gamma = 90°$
2	Tetragonal	P, I	2	$a = b \neq c$ $\alpha = \beta = \gamma = 90°$
3	Orthorhombic	P, I, F, C	4	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90°$
4	Monoclinic	P, C	2	$a \neq b \neq c$ $\alpha = \gamma = 90° \neq \beta$
5	Triclinic	P	1	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma$
6	Rhombohedral (Trigonal)	P	1	$a = b = c$ $\alpha = \beta = \gamma \neq 90°$
7	Hexagonal	P	1	$a = b \neq c$ $\alpha = \beta = 90°$ $\gamma = 120°$

4.6 Structure and packing fractions of simple cubic [SC] structure

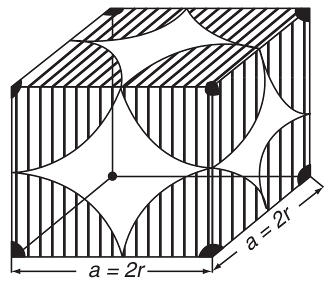
The unit cell edge lengths of this structure along the crystallographic axes and interaxial angles are equal [i.e., $a = b = c$ and $\alpha = \beta = \gamma = 90°$]. Atoms are present only at the corners of this unit cell. A corner atom is shared by eight unit cells, so that the contribution of a corner atom to a unit cell is 1/8. The cube has eight corners, hence the contribution of eight corner atoms to a unit cell or the number of atoms per unit cell = $1/8 \times 8 = 1$. Let 'r' be the radius of an atom. The surfaces of the atoms touch along the cube edges. So, the distance between the centres of two neighbouring atoms or the nearest neighbour distance ($2r$) is equal to the lattice constant 'a'. In simple cubic cell, the number of nearest neighbour atoms to an atom or co-ordination number is six. Since atoms are present at a distance of 'a' along $\pm X$, $\pm Y$ and $\pm Z$ directions. The number of nearest equidistant neighbouring atoms to an atom in the structure is called co-ordination number. Figure. 4.12 shows the simple cubic structure. Next, we find the fraction of the unit cell volume occupied by the atoms. The simple cubic structure contains only one atom per unit cell.

The volume occupied by atoms in the unit cell $(v) = 1 \times (4/3)\pi r^3$ and

The volume of unit cell (V) = a^3. Hence, the packing factor or density of packing in the unit cell (PF) = $\frac{v}{V}$

$$= \frac{(4/3)\pi r^3}{a^3} = \frac{4}{3}\frac{\pi r^3}{(2r)^3} = \frac{\pi}{6} = 0.52 \text{ or } 52\%$$

Figure 4.12 Simple cubic structure

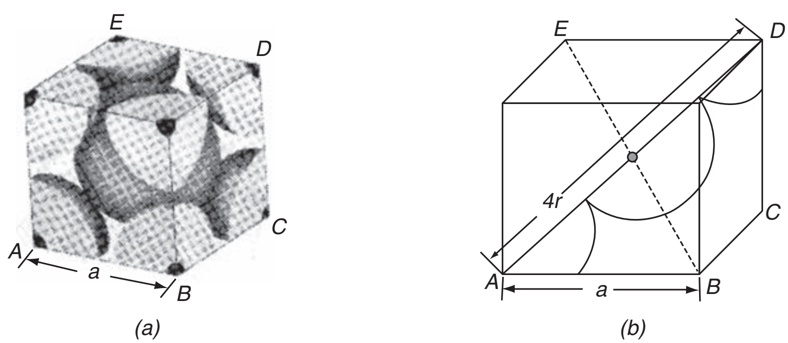


Atomic packing factor is defined as the fraction of the space occupied by atoms in the unit cell or it is the ratio of the volume occupied by atoms in the unit cell to the unit cell volume. An example for simple cubic structure is polonium crystal.

4.7 Structure and packing fractions of body-centred cubic [BCC] structure

For this unit cell, atoms are present at the corners of the cube and one atom is completely present at the centre of the unit cell. The centre of the unit cell is defined as the intersecting point of two body diagonals [AD and BE as shown in Fig. 4.13]. A corner atom is shared by eight unit cells so that the contribution of a corner atom to a unit cell is 1/8. Therefore, the number of atoms per unit cell = $(1/8)\times 8+1=2$. The centre atom is surrounded by eight corner atoms, so the coordination number is 8. The surfaces of unit cell corner atoms may not touch, but they are in contact with the centre atom, i.e. the surfaces of atoms are in contact along a body diagonal of the unit cell. Half the distance between the centres of a corner atom and central atom is equal to the radius (r) of an atom. The relation between unit cell edge length (a) and radius (r) of an atom can be obtained with reference to Fig. 4.13(b).

Figure 4.13 Body-centred cubic structure



The length of the body diagonal AD = $4r$

$\therefore\ AD^2 = AC^2 + CD^2 = AB^2 + BC^2 + CD^2 = a^2 + a^2 + a^2 = 3a^2$

$$(4r)^2 = 3a^2$$

$$4r = \sqrt{3}a$$

$$\text{(or)} \qquad a = \frac{4}{\sqrt{3}}r$$

Lastly, Packing factor (PF) $= \dfrac{\text{volume of all atoms in unit celll}}{\text{volume of unit cell}} = \dfrac{v}{V}$

$$= \frac{2 \times \frac{4}{3}\pi r^3}{a^3} = \frac{8\pi r^3}{3a^3} = \frac{8\pi r^3 3\sqrt{3}}{3(4r)^3}$$

$$= \frac{\sqrt{3}\pi}{8} = 0.68 \text{ or } 68\%.$$

The elements like tungsten, chromium, sodium, potassium, etc. possess BCC structure.

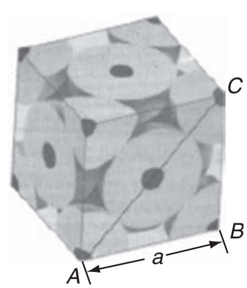
4.8 Structure and packing fractions of face-centred cubic [FCC] structure

Atoms are present at the corners and at the face centres of this cubic structure. The intersection of face diagonals represent face centre of the cube. A corner atom is shared by eight unit cells and a face-centred atom is shared by two unit cells. The cube has eight corners and bounded by six faces; so, the number of atoms per unit cell $= \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$.

Let r be the radius of an atom. The surfaces of atoms do not touch along unit cell edges but the surfaces of atoms along face diagonals of this structure are in contact. The unit cell structure is shown in Fig. 4.14. Half of the nearest neighbour distance along the face diagonal is equal to radius of an atom.

The relation between the radius of an atom and unit cell edge length of a unit cell can be obtained with reference to Fig. 4.14.

Figure 4.14 FCC structure



$$AC^2 = AB^2 + BC^2$$

$$(4r)^2 = a^2 + a^2 = 2a^2$$

$$4r = \sqrt{2}a \quad \text{or} \quad a = 2\sqrt{2}r$$

The co-ordination number is 12, and this can be explained in following way:
A face-centred atom of the cubic structure is surrounded by four corner atoms of the face of a unit cell, four surrounding face-centred atoms of the unit cell and four surrounding face-centred atoms of the adjacent unit cell. Therefore, the co-ordination number is 12. The packing factor PF of the unit cell:

$$= \frac{\text{volume occupied by all the atoms in a unit cell}}{\text{unit cell volume}}$$

$$= \frac{v}{V} = \frac{4 \times (4/3)\pi r^3}{a^3} = \frac{16\pi r^3}{3\left(2\sqrt{2}r\right)^3}$$

$$= \frac{16\pi r^3}{3 \times 8 \times 2\sqrt{2}r^3} = \frac{\pi}{3\sqrt{2}} = 0.74 \text{ or } 74\%$$

The packing factor of FCC structure is 74%. Examples for this structure are Cu, Ag, Al, etc.

4.9 Calculation of lattice constant

The unit cell edge length of a cubic system is calculated using the density of the crystal. Let 'a' be the edge length (or primitive) of a cubic unit cell and 'ρ' be the density of the crystal.

The mass of the unit cell $= \rho a^3$ __________ (4.1)

Let 'M' be the molecular weight and N_A be the Avogadro number (i.e., number of molecules per kg mole of the substance) of the crystal.

Then, mass of each molecule $= \dfrac{M}{N_A}$

If each unit cell contains n molecules (or lattice points),

Then the mass of unit cell $= n\dfrac{M}{N_A}$ __________ (4.2)

From Equations (4.1) and (4.2), we have:

$$\rho a^3 = n\frac{M}{N_A}$$

$$a^3 = \frac{nM}{\rho N_A} \quad \text{or} \quad a = \left(\frac{nM}{\rho N_A}\right)^{1/3}$$

Formula

1. $$a = \left[\frac{nM}{\rho N_A}\right]^{1/3}$$

Solved Problems

1. *Chromium has BCC structure. Its atomic radius is 0.1249 nm. Calculate the free volume/unit cell.*

(Set-4–May 2007), (Set-4–Sept. 2006)

Sol: Given data are

Atomic radius of chromium, $r = 0.1249$ nm.

Free volume/unit cell = ?

If 'a' is the BCC unit cell edge length, then the relation between 'a' and 'r' is

$$a = \frac{4}{\sqrt{3}} r = \frac{4}{\sqrt{3}} \times 0.1249 \text{ nm}$$

$$= 0.28845 \text{ nm.}$$

Volume of unit cell, $V = a^3 = (0.28845)^3 \text{ nm}^3$

$$= 0.024 \text{ nm}^3$$

Number of atoms in BCC unit cell = 2

Hence volume of atoms in unit cell, $v = \frac{4}{3}\pi r^3 \times 2 = 0.01633 \text{ nm}^3$

Free volume/unit cell = $V - v = 0.00767 \text{ nm}^3$

2. *Lithium crystallizes in BCC structure. Calculate the lattice constant, given that the atomic weight and density for lithium are 6.94 and 530 kg/m³ respectively.*

(Set-4–Nov. 2003)

Sol: Lithium crystallizes in BCC structure, so the number of atoms per unit cell, $n = 2$

Atomic weight, $M = 6.94$

Density, $\rho = 530 \text{ Kg/m}^3$

Lattice constant, a = ?

$$a^3 = \frac{nM}{\rho N_A}, \text{ where } N_A = \text{Avogadro's number}$$

$$a^3 = \frac{2 \times 6.94}{530 \times 6.02 \times 10^{26}}$$

$$= 43.50 \times 10^{-30}$$

$$\therefore \; a = 3.517 \times 10^{-10} \text{ m}$$

$$= 3.517 \text{ Å}$$

3. *Iron crystallizes in BCC structure. Calculate the lattice constant, given that the atomic weight and density of iron are 55.85 and 7860* kg/m^3*, respectively.*

(Set-3–Sept. 2006), (Set-1–Nov. 2003)

Sol: Atomic weight of iron, $M = 55.85$

Density of iron, $\rho = 7860\ kg/m^3$

Find lattice constant, a in BCC iron

Number of atoms in BCC unit cell, $n = 2$

We know that:

$$a = \left(\frac{nM}{\rho N_A}\right)^{1/3}$$

$$\left[\frac{2 \times 55.85}{7860 \times 6.02 \times 10^{26}}\right]^{1/3} = 2.87 \times 10^{-10}\ m = 2.87\ Å$$

4. *If the edge of the unit cell of a cube in the diamond structure is 0.356* nm*, calculate the number of* atoms/m^3.

(Set-3–Nov. 2003)

Sol: The lattice constant of diamond, $a = 0.356\ nm = 0.356 \times 10^{-9}\ m$

The number of carbon atoms per unit cell, $n = 8$

The number of unit cells in 1 $m^3 = \dfrac{1\ m^3}{a^3}$

and the number of atoms per $m^3 = \dfrac{1\ m^3}{a^3} \times 8 = \dfrac{1\ m^3 \times 8}{(0.356 \times 10^{-9}\ m)^3}$

$$= \frac{8 \times 10^{27}}{(0.356)^3} = 177.3 \times 10^{27}\ atoms/m^3$$

5. *A metal in BCC structure has a lattice constant 3.5* Å. *Calculate the number of atoms per sq. mm area in the (200) plane.*

Sol: Lattice constant, $a = 3.5$ Å

The (200) plane is perpendicular to X-axis and passes through the centre of the unit cell. So, this plane contains only the central atom.

$\therefore$ The area per atom $= a^2 = 3.57\ Å \times 3.5\ Å = 12.25\ Å^2$

Number of atoms per sq. mm $= \dfrac{1\ mm \times 1\ mm}{12.25\ Å^2}$

$$= \frac{10^7\ Å \times 10^7\ Å}{12.25\ Å^2} = 8.16 \times 10^{12} \qquad [\text{since } 1\ mm = 10^7\ Å]$$

6. *Germanium crystallizes in diamond (form) structures with 8 atoms per unit cell. If the lattice constant is 5.62* Å, *calculate its density.*

Sol: Number of atoms per unit cell, $n = 8$

Lattice constant, $a = 5.62$ Å $= 5.62 \times 10^{-10}$ m

Atomic weight of Ge, $M = 72.59$

Density, $\rho = ?$

We know that $a^3 = \dfrac{nM}{\rho N_A}$, where N_A = Avogadro's number

$$\rho = \frac{nM}{a^3 N_A}$$

$$= \frac{8 \times 72.59}{\left[5.62 \times 10^{-10}\right]^3 \times 6.02 \times 10^{26}} \text{ Kg/m}^3$$

$$= 5434.5 \text{ kg/m}^3.$$

Multiple-choice Questions

1. If the atoms or molecules in a solid are periodical at regular intervals of distances in three dimensions, then that solid is known as __________ .

   (a) crystalline solid (b) amorphous solid
   (c) liquid crystals (d) none

2. Unit cells for most of the crystals are __________ .

   (a) spherical (b) elliptical (c) parallelopiped (d) none

3. Crystallographic axes are obtained by the intersection of __________ non-coplanar faces of the unit cell.

   (a) three (b) four (c) five (d) six

4. The number of crystal systems is __________ .

   (a) 5 (b) 7 (c) 14 (d) 21

5. The number of Bravais lattices is __________ .

   (a) 256 (b) 7 (c) 14 (d) 37

6. A cubic crystal system is represented by __________ .

   (a) $a = b = c$, $\alpha = \beta = \gamma \neq 90°$ (b) $a = b \neq c$, $\alpha = \beta = \gamma = 90°$ (c) $a = b = c$, $\alpha = \beta = \gamma = 90°$ (d) $a \neq b \neq c$, $\alpha = \beta = \gamma = 90°$

7. Orthorhombic crystal system is represented by __________ .

   (a) $a = b = c$, $\alpha = \beta = \gamma = 90°$ (b) $a \neq b \neq c$, $\alpha = \beta = \gamma = 90°$ (c) $a \neq b \neq c$, $\alpha = \beta = \gamma \neq 90°$ (d) $a \neq b \neq c$, $\alpha \neq \beta \neq \gamma \neq 90°$

8. Tetragonal crystal system is represented by __________ .

   (a) $a = b \neq c$, $\alpha = \beta = \gamma = 90°$ (b) $a \neq b \neq c$, $\alpha = \beta = \gamma = 90°$ (c) $a \neq b = c$, $\alpha = \beta = \gamma \neq 90°$ (d) $a = b = c$, $\alpha = \beta = \gamma = 90°$

9. Monoclinic crystal system is represented by ___________ .

(a) $a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$ (b) $a \neq b \neq c$ $\alpha = \gamma = 90°$ $\beta \neq 90°$ (c) $a = b = c$ $\alpha = \gamma = 90°$ $\beta = 90°$ (d) $a \neq b = c$ $\alpha = \gamma = 90°$ $\beta = 90°$

10. Triclinic crystal system is represented by ___________ .

(a) $a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$ (b) $a \neq b = c$ $\alpha \neq \beta \neq \gamma \neq 90°$ (c) $a = b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$ (d) $a \neq b \neq c$ $\alpha = \beta = \gamma \neq 90°$

11. Rhombohedral [Trigonal] system is represented by ___________ .

(a) $a = b \neq c$ $\alpha = \beta = \gamma = 90°$ (b) $a = b = c$ $\alpha = \beta = \gamma \neq 90°$ (c) $a = b = c$ $\alpha = \beta \neq \gamma = 90°$ (d) $a \neq b = c$ $\alpha = \beta = \gamma \neq 90°$

12. Hexagonal crystal system is represented by ___________ .

(a) $a = b \neq c$ $\alpha = \beta = \gamma = 90°$ (b) $a = b = c$ $\alpha = \beta = 90°$ $\gamma = 120°$ (c) $a = b \neq c$ $\alpha = \beta = 90°$ $\gamma = 120°$ (d) $a = b \neq c$ $\alpha = \beta = 120°$ $\gamma = 90°$

13. The number of atoms per unit cell of BCC structure is ___________ .

(a) 1 (b) 2 (c) 3 (d) 4

14. In body-centred cubic structure, the length of unit cell edge interms of radius of atom (r) is ___________ .

(a) $\frac{4}{3}r$ (b) $\frac{4}{\sqrt{3}}r$ (c) $\frac{\sqrt{4}}{3}r$ (d) $\frac{4}{3}\sqrt{r}$

15. The packing factor of BCC structure is ___________ %.

(a) 68 (b) 52 (c) 74 (d) 46

16. The packing factor of face-centred cubic structure is ___________ %.

(a) 68 (b) 52 (c) 74 (d) 46

17. When the periodicity of atoms or molecules is extended throughout the solid, then it is known as ___________ solid.

(a) single crystalline (b) polycrystalline
(c) amorphous (d) none

18. If the periodicity of atoms or molecules is extended in large number of small regions of different sizes in the solid, then it is known as ___________ .

(a) single crystalline solid (b) polycrystalline solid
(c) amorphous solid (d) none

19. The study of geometric form and other physical properties of crystalline solids by using X-rays, electron beam and neutron beam constitute ___________ .

(a) spectroscopy (b) physiotherapy
(c) crystallography (d) none

20. If an atom or a unit of atoms in a crystal is replaced by a point in space, then it results points in space is called ___________ .

(a) space lattice (b) crystal symmetry
(c) spectrum (d) diffraction

21. If the surroundings of each lattice point is the same or the lattice points are identical, then such a lattice is called ___________ .
   (a) Bravais lattice (b) space lattice
   (c) Braggs lattice (d) none
22. The arrangement of atoms in a crystal is called ___________ .
   (a) lattice (b) crystal structure
   (c) crystal symmetry (d) none
23. The number of Bravais lattices in cubic crystal system is ___________ .
   (a) one (b) two
   (c) three (d) four
24. The number of Bravais lattices in orthorhombic crystal system is ___________ .
   (a) one (b) two (c) three (d) four
25. The number of Bravais lattices in tetragonal and monoclinic systems is ___________ .
   (a) equal (b) unequal (c) both a and b (d) none
26. The packing factor of simple cubic structure is ___________ .
   (a) 68% (b) 74% (c) 52% (d) 34%
27. If the number of lattice points per unit cell is one, then it is called ___________ unit cell.
   (a) primitive (b) non-primitive (c) both a and b (d) none
28. The number of atoms per unit cell of face-centred cubic structure is ___________ .
   (a) one (b) two (c) three (d) four

Answers

1. a	2. c	3. a	4. b	5. c	6. c	7. b	8. a	9. b	10. a
11. b	12. c	13. b	14. b	15. a	16. c	17. a	18. b	19. c	20. a
21. a	22. b	23. c	24. d	25. a	26. c	27. a	28. d		

Review Questions

1. Show that FCC is the most closely packed of the three cubic structures by working out the packing factors.
   (Set-1, Set-3–May 2007), (Set-3–Sept. 2007), (Set-2–June 2005), (Set-3–Nov. 2004), (Set-4–May 2004)
2. Explain the terms: (i) basis (ii) space lattice and (iii) unit cell.
   (Set-4–May 2006), (Set-1–Sept. 2007), (Set-1, Set-2–June. 2005), (Set-1–Nov. 2004), (Set-3–May 2003)
3. Describe seven crystal systems with diagrams.
   (Set-1–Sept. 2007), (Set-4–May 2007), (Set-4–May 2006), (Set-4–Sept. 2006), (Set-1, Set-2–June 2005), (Set-1–Nov. 2004), (Set-3–May 2003)
4. Obtain the relations between the edge of the unit cell and atomic radius for the BCC and FCC lattices.
   (Set-4–Nov. 2003)
5. What are Bravais lattices? (Set-3–Sept. 2006), (Set-1–Nov. 2003)

6. Deduce packing factors for simple cubic and BCC structures. (Set-3–Sept. 2006), (Set-1–Nov. 2003)
7. Define co-ordination number and packing factor of a crystal.
(Set-1–Sept. 2008), (Set-1, Set-3–May 2007), (Set-2–Sept. 2006), (Set-2–Nov. 2003)
8. Describe FCC crystal structure.
(Set-2–Sept. 2006), (Set-1, Set-3–May 2006), (Set-2–Nov. 2003)
9. Obtain an expression for the packing factor of FCC structure.
(Set-1–Sept. 2008), (Set-2–Sept. 2006), (Set-1, Set-3–May 2006), (Set-2–Nov. 2003)
10. What is packing fraction? Calculate the packing fraction for a BCC lattice. (Set-3–Nov. 2003)
11. Define crystal lattice, unit cell, lattice parameter and coordination number.
(Set-1–May 2007), (Set-1–Sept. 2006)
12. Explain the unit cell and lattice parameters. What is a primitive cell and how does it differ from unit cell.
(Set-4–May 2007), (Set-4–Sept. 2006)
13. Consider a body centred cubic lattice of identical atoms having radius 'R' compute (i) the number of atoms per unit cell (ii) The coordination number and (iii) the packing fraction.
(Set-1–Sept. 2006), (Set-1–May 2007)
14. Explain the terms: (i) basis, (ii) space lattice, (iii) lattice parameters and (iv) unit cell. (Set-3–Sept. 2008)
15. Describe BCC structure, with suitable example. (Set-1–Sept. 2008)
16. Describe in detail, the seven crystal systems with diagrams. (Set-4–Sept. 2008)
17. Prove that which type of the cubic crystal structure has closest packing of atoms. Describe the relation between the atomic radius and the unit cell dimension of the crystal, mentioned above.
(Set-2–Sept. 2007)
18. Tabulate the characteristics of the unit cells of different crystal systems.
19. Illustrate Bravais lattices.
20. Illustrate simple cubic, FCC and BCC crystal structures.
21. What is space lattice? Find the packing fraction for BCC and FCC crystals.
22. Show that FCC crystals are closely packed than BCC crystals.
23. Classify various lattice types in the crystal system.
24. What is a Bravais lattice? What are the different space lattices in the cubic system?

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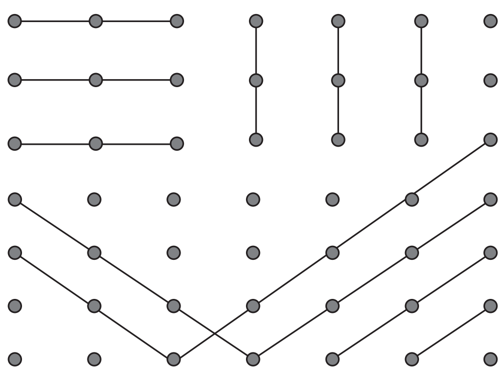
CHAPTER 5

X-Ray Diffraction

5.1 Crystal planes, directions and Miller indices

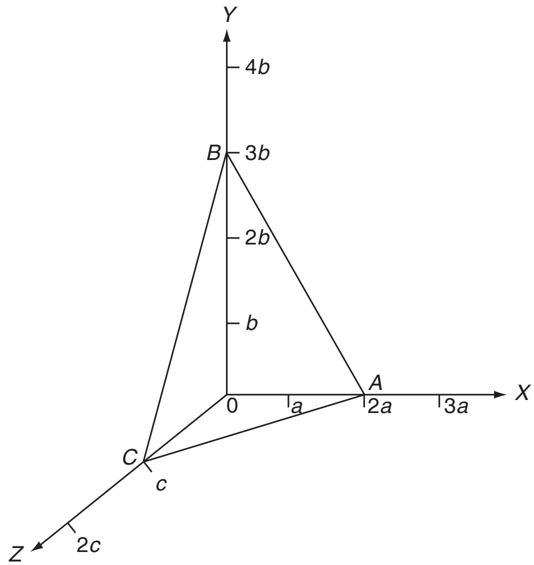
Crystal planes are defined as some imaginary planes inside a crystal in which large concentration of atoms are present. Inside the crystal, there exists certain directions along which large concentration of atoms exists. These directions are called crystal directions. Figure 5.1 shows a two-dimensional lattice with different orientations of crystal planes.

Figure 5.1 A two-dimensional lattice with crystal planes



Crystal planes and directions can be represented by a set of three small integers called Miller indices [because Miller derived a method of representing crystal planes]. These integers are represented in general as *h*, *k* and *l*. If these integers are enclosed in round brackets as (*hkl*), then it represents a plane. On the other hand, if they are enclosed in square brackets as [*hkl*], then it represents crystal direction perpendicular to the above-said plane. Next, we will see the way of obtaining Miller indices for a plane.

Figure 5.2 Miller indices for a plane *ABC*



(i) As shown in Fig. 5.2, take a lattice point as origin '0' of crystallographic axes X, Y and Z in a space lattice. The unit cell translational distances or lattice constants along X, Y and Z directions are a, b and c, respectively. Let a crystal plane ABC intersect these three axes at $2a$, $3b$ and c. In general, the intercepts can be represented as pa, qb, and rc.

(ii) Divide these intercepts with lattice points translational distances along the axes to obtain intercepts of the plane in terms of multiples of unit cell translational lengths.

i.e., $\dfrac{2a}{a}, \dfrac{3b}{b}, \dfrac{c}{c}$

$2, \quad 3, \quad 1$

in general $\dfrac{pa}{a}, \dfrac{qb}{b}, \dfrac{rc}{c}$

$p, \quad q, \quad r$

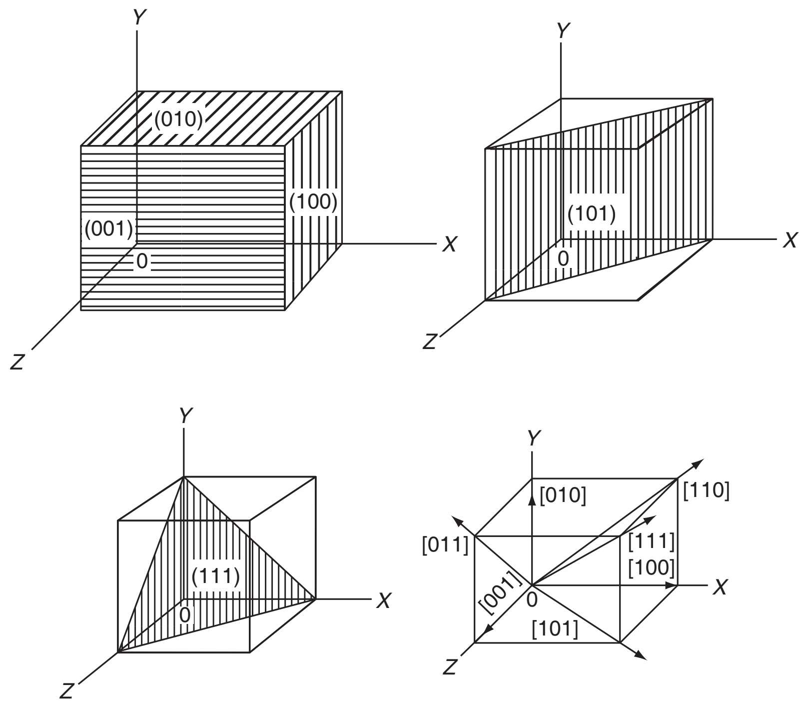
(iii) Take the reciprocals of these multiples, they are $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{1}$; in general $\dfrac{1}{p}, \dfrac{1}{q}, \dfrac{1}{r}$

(iv) Clear these fractions [by multiplying with LCM] to smallest integers having the same ratio as the fractions, enclose these integers in brackets.

$\dfrac{1}{2}\times 6, \dfrac{1}{3}\times 6, \dfrac{1}{1}\times 6$

$3 \qquad 2 \qquad 6$

Figure 5.3 Represent some important crystal planes and directions in a cubic crystal



in general $\dfrac{pqr}{p}, \dfrac{pqr}{q}, \dfrac{pqr}{r}$

$qr \quad pr \quad pq$

Miller indices of the plane *ABC* is (326). In general, indices of a plane are represented as $(hkl) = (qr\ pr\ pq)$

or $\dfrac{1}{p}:\dfrac{1}{q}:\dfrac{1}{r} = h:k:l$

Miller indices may be defined as a set of three small integers obtained by clearing the reciprocals of the three intercepts [in terms of multiples of unit cell edges] made by a plane on crystallographic axes.

Now, we will see the important features of Miller indices:

(i) Miller indices represent a set of equidistant parallel planes.
(ii) If the Miller indices of a plane represent some multiples of Miller indices of another plane, then these planes are parallel. For example (844) and (422) or (211) are parallel planes.
(iii) If (*hkl*) are the Miller indices of a plane, then the plane divides the lattice constant '*a*' along *X*-axis into *h* equal parts, '*b*' along *Y*-axis into *k* equal parts and '*c*' along *Z*-axis into *l* equal parts.

(iv) If a plane is parallel to one of the crystallographic axes, then the plane intersects that axis at infinity and the Miller indices along that direction is zero.

(v) If a plane cuts an axis on the negative side of the origin, then the corresponding index is negative and is indicated by placing a minus sign above the index. For example, if the plane cuts on negative *Y*-axis, then Miller indices of the plane is $(h\bar{k}l)$.

(vi) When Miller indices are enclosed in curly brackets, $\{hkl\}$, they refer to planes which in the crystal are equivalent even though their Miller indices may differ. For example in a cubic lattice, all cube faces are equivalent, they are (100), (010), (001), $(\bar{1}00), (0\bar{1}0), (00\bar{1})$; these planes are represented as {100}. Similarly, a full set of equivalent directions in a crystal is represented by a symbol $<hkl>$. For example, the eight body diagonals of a cube $[111], [\bar{1}\bar{1}\bar{1}], [\bar{1}11], [1\bar{1}1], [11\bar{1}], [\bar{1}\bar{1}1], [\bar{1}1\bar{1}], [1\bar{1}\bar{1}]$ are designated as $<111>$.

5.2 Distance of separation between successive *hkl* planes

The separation between successive parallel planes in rectangular axes crystal system can be extracted easily. Let us consider a rectangular [Cartesian] coordinate system with origin '0'at one of the lattice points. Let (hkl) be the Miller indices of a plane *ABC*, which makes intercepts *OA*, *OB* and *OC* on *X*, *Y* and *Z* axes, respectively as shown Fig 5.4. A normal to this plane from the origin passes through a point *N* in the plane *ABC*, such that $ON = d_1$. This normal makes α', β', and γ' angles with *X*, *Y* and *Z*-axes, respectively. Since the plane segments '*a*' into '*h*' equal parts, *b* into *k* equal parts and *c* into *l* equal parts, then the intercepts *OA*, *OB* and *OC* are such that:

$$OA = \frac{a}{h},\ OB = \frac{b}{k}\text{a} \qquad \text{and} \qquad OC = \frac{c}{l} \qquad \text{\_\_\_\_\_\_\_\_\_\_} \ (5.1)$$

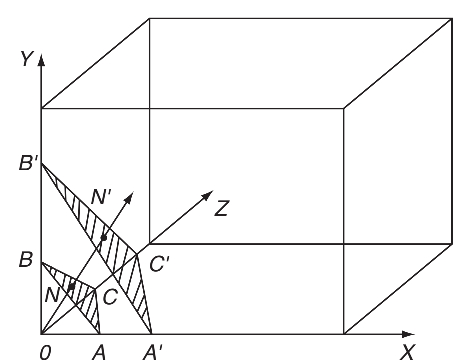
where *a*, *b*, *c* are the unit cell edge lengths along *X*, *Y* and *Z*-axes, respectively.

From Fig. 5.4

$$\cos\alpha' = \frac{d_1}{OA},\ \cos\beta' = \frac{d_1}{OB}\text{a} \qquad \text{and} \qquad \cos\gamma' = \frac{d_1}{OC} \qquad \text{\_\_\_\_\_\_\_\_\_\_} \ (5.2)$$

Let the coordinates of *N* be *x*, *y* and *z* along *X*, *Y* and *Z* axes, then:

Figure 5.4 Orthorhombic crystal



$$(ON)^2 = d_1^2 = x^2 + y^2 + z^2 \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.3)$$

Also from Fig. 5.4:

$$\cos\alpha' = \frac{x}{d_1}, \ \cos\beta' = \frac{y}{d_1}, \ \cos\gamma' = \frac{z}{d_1} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.4)$$

Substituting Equation (5.4) in (5.3) gives

$$d_1^2 = d_1^2\cos^2\alpha' + d_1^2\cos^2\beta' + d_1^2\cos^2\gamma'$$

$$= d_1^2\,[\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma']$$

(or) $\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma' = 1$ __________ (5.5)

Substituting Equation (5.2) in (5.5) gives

$$\frac{d_1^2}{(OA)^2} + \frac{d_1^2}{(OB)^2} + \frac{d_1^2}{(OC)^2} = 1 \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.6)$$

Again substitute Equation (5.1) in (5.6)

$$\frac{d_1^2\,h^2}{a^2} + \frac{d_1^2 k^2}{b^2} + \frac{d_1^2 l^2}{c^2} = 1 \qquad \text{(or)} \qquad d_1^2\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right] = 1$$

$$d_1 = \frac{1}{\sqrt{\dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} + \dfrac{l^2}{c^2}}} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.7)$$

Let $\left(\dfrac{h}{2}\,\dfrac{k}{2}\,\dfrac{l}{2}\right)$ be the Miller indices of the next plane $A'\ B'\ C'$, this plane makes intercepts OA', OB' and OC' on X, Y and Z axes, respectively. A normal from the origin to this plane passes through a point N', so that $ON' = d_2$. As the extension of d_1 is d_2, it makes same angles α', β' and γ' with X, Y and Z-axes, respectively. Since the plane segments 'a' into $h/2$ equal parts, b into $k/2$ equal parts and c into $l/2$ equal parts, then the intercepts OA', OB' and OC' are such that:

$$OA' = \frac{a}{\left(h/2\right)} = \frac{2a}{h}, \ OB' = \frac{b}{\left(k/2\right)} = \frac{2b}{k} \qquad \text{and} \qquad OC' = \frac{c}{\left(l/2\right)} = \frac{2c}{l} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.1')$$

From Fig. 5.4,

$$\cos\alpha' = \frac{d_2}{(OA')}, \ \cos\beta' = \frac{d_2}{(OB')} \qquad \text{and} \qquad \cos\gamma' = \frac{d_2}{(OC')} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.2')$$

Let the coordinates of N' are x', y' and z' along X, Y and Z-axes, respectively.

$$\left(ON'\right)^2 = d_2^2 = x'^2 + y'^2 + z'^2 \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.3')$$

Also from Fig 5.4:

$$\cos\alpha' = \frac{x'}{d_2},\ \cos\beta' = \frac{y'}{d_2} \quad \text{and} \quad \cos\gamma' = \frac{z'}{d_2} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.4')$$

Substituting Equation (5.4′) in (5.3′) gives

$$d_2^2 = d_2^2\cos^2\alpha' + d_2^2\cos^2\beta' + d_2^2\cos^2\gamma'$$

$$= d_2^2\left[\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma'\right]$$

(or) $\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma' = 1$ __________ (5.5′)

Substituting Equation (5.2′) in (5.5′) gives

$$\frac{d_2^2}{(OA')^2} + \frac{d_2^2}{(OB')^2} + \frac{d_2^2}{(OC')^2} = 1 \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.6')$$

Again substitute Equation (5.1′) in (5.6′) gives:

$$\frac{d_2^2h^2}{(2a)^2} + \frac{d_2^2k^2}{(2b)^2} + \frac{d_2^2l^2}{(2c)^2} = 1 \quad \text{(or)} \quad d_2^2\left[\frac{h^2}{(2a)^2} + \frac{k^2}{(2b)^2} + \frac{l^2}{(2c)^2}\right] = 1$$

$$\text{(or)} \quad d_2 = \frac{1}{\sqrt{\frac{h^2}{(2a)^2} + \frac{k^2}{(2b)^2} + \frac{l^2}{(2c)^2}}} \quad \text{(or)} \quad d_2 = \frac{2}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.7')$$

Let the separation between the planes *ABC* and *A′B′C′* is ‘*d*’.

$$\therefore\ d = d_2 - d_1 = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.8)$$

Using Equation (5.8), we can determine the interplanar separation in orthorhombic crystals.
For tetragonal crystal $a = b \neq c$, substitute these values in Equation (5.8), we have:

$$d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{a^2} + \frac{l^2}{c^2}}} = \frac{1}{\sqrt{\frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}}} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.9)$$

For cubic crystals: $a = b = c$, substitute these values in Equation (5.8), we have:

$$d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{a^2} + \frac{l^2}{a^2}}} \quad \text{(or)} \quad d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad \text{\_\_\_\_\_\_\_\_\_\_} (5.10)$$

The calculation of interplanar spacing for other crystal systems is complicated, so we will not discuss them.

5.3 Imperfections in crystals

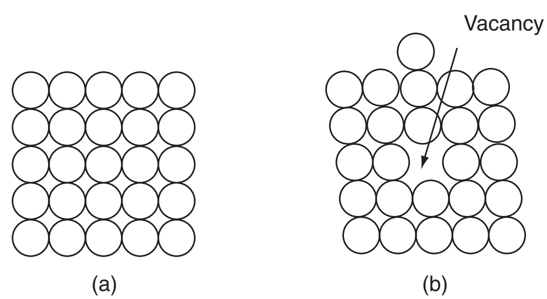
In a sound crystal (or in an ideal crystal), the atoms are arranged regularly and periodically in three dimensions. But the grown crystals [or real crystals] may contain imperfections or defects. These defects are mainly divided into point, line, surface and volume defects. We will discuss point and line defects. They are described below:

(1) Point defects: As the name indicates, these defects are at some points in the crystal. So, these are also called zero-dimensional defects. The point defects are divided into three categories: (a) lattice site defects; this includes vacancies [Schottky defect] and interstitialcies [Frenkel defect], (b) compositional defects; this includes substitutional impurity and interstitial impurity and (c) electronic defects. These defects are discussed below.

(a) Lattice site defects: In this type of defects, some atoms may not be present in their regular atomic sites. They are:

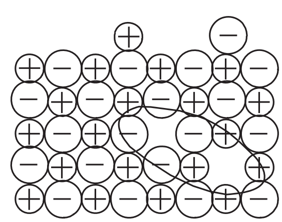
(i) Vacancies: As shown in Fig. 5.5, at a lattice point, one or two or three atoms are missed, and this is referred to as single or double or triple vacancies, respectively. The vacancies are formed due to the imperfect packing during crystallization or due to thermal vibrations at high temperatures.

Figure 5.5 (a) Perfect crystal and (b) vacancy defect



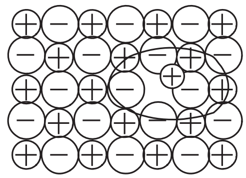
Schottky defect: In ionic crystals, if a cation vacancy exists, then in the very nearby place an anion vacancy also exists. i.e., usually an anion and cation pair is moved to the surface of the crystal, so that charge neutrality is maintained in the vacancy region as shown in Fig. 5.6. This is known as Schottky defect. Crystals such as NaCl, KCl, KBr, etc. show Schottky defect.

Figure 5.6 Schottky defect



(ii) Interstitial defect: If an atom is moved to an interstitial space in the crystal, then the defect is known as interstitial defect.

Figure 5.7 Frenkel defect

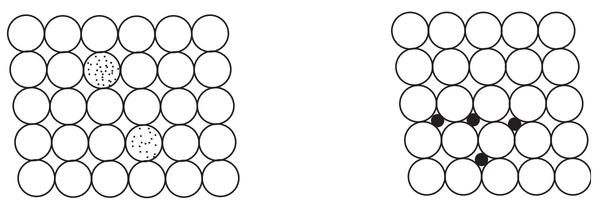


Frenkel defect: In ionic crystals, if a cation [positive ion] moves to an interstitial space, then a vacancy is formed in its atomic position. Here, charge neutrality is maintained in the defective region as shown in Fig. 5.7. This type of defect is known as Frenkel defect. Crystals such as CaF_2, AgBr, AgI, etc. show Frenkel defect.

(b) Compositional defect: The presence of impurity atoms in the crystal leads to compositional defects. Impurity atoms are present at the sites of regular parent atoms or in the interstitial spaces. These defects are described below.

(i) Substitutional defect: As shown in Fig. 5.8(a), during crystallization few foreign atoms occupy the regular parental atoms sites. For example, in extrinsic semiconductors either third or fifth group atoms occupy the sites of silicon or germanium atoms.

Figure 5.8 (a) Substitutional defect and (b) interstitial defect



(a) (b)

(ii) Interstitial impurity defect: The spaces between the parental atoms in a crystal are known as interstitial spaces. Small-sized [lower atomic number] atoms, such as hydrogen, etc. may fit into these interstitial spaces. These atoms are known as interstitial atoms and the defect formed due to the presence of interstitial atoms is known as interstitial defect. This is shown in Fig. 5.8(b). If 'r' is the radius of a parent atom, then a octahedral and a tetrahedral space can accommodate an interstitial atom of radius $0.414r$ and $0.225r$, respectively.

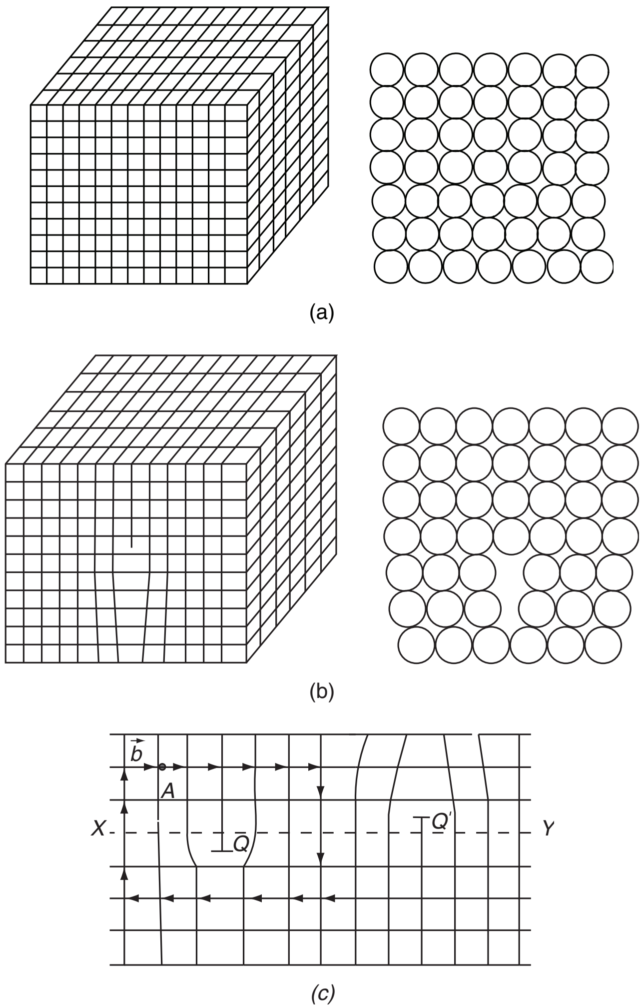
(c) Electronic defects: Non-uniformity of charge or energy distribution in the crystal is referred to as electronic defect. The presence of impurity atoms such as substitutional and interstitial atoms and vacancies can vary the uniform distribution of electronic charge in the crystal. So, the presence of these defects also leads to electronic defects. In semiconductors, temperature variation changes charge concentration, so the variation of temperature [i.e., thermal energy] leads to electronic defects.

Point defects are formed by thermal fluctuations, by severe deformation [i.e., by hammering or rolling] and by bombarding with high energetic particles.

(2) Line defects: If a crystal plane ends somewhere in the crystal, then along the edge of that incomplete plane produces defect in the crystal called line defect. The line defect is of two types: they are (i) edge dislocation and (ii) screw dislocation. These are described below.

(i) Edge dislocation: Figure 5.9(a) shows three-dimensional view and front face of a perfect crystal. The vertical crystal planes are parallel to side faces of a crystal is shown in the figure. One of the crystal planes does

Figure 5.9 (a) Three-dimensional view of perfect crystal; front view of perfect crystal; (b) three-dimensional view of edge dislocation crystal; front view of edge dislocation crystal and (c) positive and negative edge dislocations

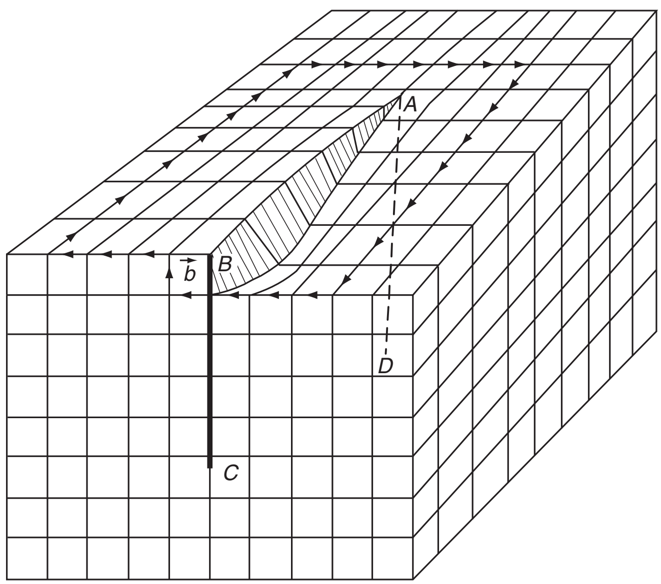


not pass from top to bottom face of the crystal, but ends some where in the crystal as shown in Fig. 5.9(b). In this crystal, just above the edge of incomplete plane, the atoms are in a state of compression so that the bond distances are less than normal values and below the edge of incomplete plane, the atoms are far apart, so the bond distances are larger than normal values. This situation extends all along the edge of this incomplete plane producing edge dislocation. The extra plane indicated in Fig. 5.9(b) can be either above or below the slip plane shown as dotted line *X*, *Y* in Fig. 5.9(c). If the incomplete extra plane is above the slip plane, then the edge dislocation is positive and is represented by the symbol $\perp$; on the other hand if it is below the slip plane, then the edge dislocation is negative and is represented by the symbol $\top$. If one plane of atoms glides over another separated by an integral multiple of interatomic distance is called slip, and the slip plane is the plane in which slip has taken place. Thus, the crystal consists of slipped and normal regions.

The magnitude and direction of the displacement of crystal planes due to edge dislocation can be represented by a vector called Burger's vector, which is perpendicular to the dislocation line. This indicates how much and in what direction the lattice above the slip plane is shifted with respect to the lattice below the slip plane. Figure 5.9(c) shows a method of determining Burger's vector for edge dislocation. To find the magnitude and direction of Burger's vector, one starts arbitrarily from a lattice point *A*, drawing atom-to-atom vectors round the dislocation in clockwise direction to form a closed circuit. Here, the number of vectors in horizontal direction at the top and bottom and vertical vectors at the left and right are equal, but the circuit is not closed unless we put the vector $\vec{b}$, as shown in the circuit. This is the Burger's vector for the above-said edge dislocation.

(ii) Screw dislocation: The crystal planes spiral about a line in the crystal, called dislocation line. The screw dislocation is shown in Fig. 5.10. Due to the spiralling of crystal planes, the atoms at one end of the plane are displaced by one atomic distance with respect to the other end of the plane in perpendicular direction to the plane. As shown in Fig. 5.10, the plane *ABCD* is the slipped area. The upper portion of the crystal has been sheared by one atomic distance compared to the right side region of the crystal. Slip has not taken place to the right side of *AD*, so *AD* is the dislocation line. Burger's circuit is completed around the dislocation. The Burger's vector is parallel to the dislocation line. By knowing the Burger's vector and dislocation line, the dislocation is completely described.

Figure 5.10 Screw dislocation and Burger's vector



5.4 Energy for the formation of a vacancy and number of vacancies—At equilibrium concentration

Energy supply to a crystal moves some of the atoms present at regular atomic sites in the interior of the crystal to the surface, so that vacancies are formed inside the crystal. If we supply energy to an ionic crystal, then

either cation–anion pairs are moved to the surface [Schottky defect] or cations are moved to interstitial spaces [Frenkel defect], so that vacancies are formed inside the ionic crystal. We shall find the relation between number of vacancies and energy of formation of a vacancy in all the above cases.

(i) In metallic crystals: Let a crystal contains N number of atoms. The energy required to move an atom at a regular atomic site in the interior of the crystal to the surface is E_V i.e., the energy required to create a vacancy. To create 'n' number of isolated vacant sites, the energy required is nE_V. At some thermal equilibrium temperature 'T', let 'n' number of vacancies present in the crystal. The number of ways these 'n' vacancies are created is given by (P).

$$P = \frac{N!}{(N-n)!\,n!} \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.11)}$$

The vacancies created inside the crystal produces disorder in the crystal. The disorder can be measured in terms of entropy. The increase in entropy (S) due to the increase of vacancies is:

$$S = K_B \log P \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.12)}$$

where K_B is Boltzmann constant, substituting Equation (5.11) in (5.2), we have:

$$S = K_B \log \frac{N!}{(N-n)!\,n!} \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.13)}$$

The creation of vacancies produces not only the change in entropy but also change in free energy (F) of the crystal.

$$F = U - TS \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.14)}$$

where $U = nE_V$ = internal energy of crystal at temperature TK. Equations (5.12) and (5.14) are taken from thermodynamics. Substituting Equation (5.13) in (5.14) gives:

$$F = nE_V - K_B T \log\left[\frac{N!}{(N-n)!\,n!}\right]$$

$$= nE_V - K_B T\,[\log N! - \log (N-n)! - \log n!] \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.15)}$$

The logarithmic term in the above equation can be simplified using Stirling's approximation,

$$\log x! = x \log x - x$$

Equation (5.15) becomes:

$$F = nE_V - K_B T\,[\,N \log N - (N-n) \log (N-n) - n \log n] \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.16)}$$

In thermal equilibrium at constant volume, the free energy is minimum with respect to changes in 'n'.

$$\therefore \left[\frac{\partial F}{\partial n}\right]_T = 0 = E_V - K_B T \log\left(\frac{N-n}{n}\right) \quad \text{(or)} \quad \frac{E_V}{K_B T} = \log\left[\frac{N-n}{n}\right] \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.17)}$$

Taking exponential on both sides of Equation (5.17), we have:

$$\exp\left(\frac{E_V}{K_B T}\right) = \frac{N-n}{n} \qquad \text{(or)} \qquad n = (N-n)\exp\left[\frac{-E_V}{K_B T}\right] \qquad \text{if } n << N \text{ then}$$

$$n \approx N \exp\left[\frac{-E_V}{K_B T}\right] \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.18)}$$

The above equation indicates that by decreasing equilibrium temperature, the concentration of vacancies decreases.

(b) In Ionic crystals: Here, we see Schottky and Frenkel defects separately.

(i) Schottky defect: In ionic crystals, equal number of cations [positive ions] and anions [negative ions] vacancies are formed i.e., usually cation–anion-paired vacancies are formed, so that charge neutrality is maintained in the crystal. The energy required to move a cation and an anion from interior of the crystal to the surface is E_P. At some thermal equilibrium temperature (T), let 'n' pairs of cation–anion vacancies present in a crystal containing 'N' pairs of ions. The number of ways these n-pairs of vacancies are created is given by (P).

$$P = \left[\frac{N!}{(N-n)!\,n!}\right]^2 \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.19)}$$

The vacancies created inside the crystal produces a disorder in the crystal. The disorder can be measured in terms of entropy. The increase in entropy (S), due to the creation of n pairs of vacancies is

$$S = K_B \log P \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.20)}$$

where K_B is Boltzmann constant, substituting Equation (5.19) in (5.20) we have:

$$S = K_B \log\left[\frac{N!}{(N-n)!\,n!}\right]^2 \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.21)}$$

The vacancies produce not only change in entropy but also change in free energy (F) of the crystal.

$$\therefore\ F = U - TS \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.22)}$$

where $U = n\,E_P$ = internal energy of the crystal at temperature T. Equations (5.20) and (5.22) are taken from thermodynamics; substituting Equation (5.21) in (5.22) gives:

$$F = nE_P - K_B T \log\left[\frac{N!}{(N-n)!\,n!}\right]^2$$

$$= nE_P - 2\,K_B T\,[\log N! - \log\,(N-n)\,! - \log n\,!] \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.23)}$$

The logarithmic term in the above equation can be simplified using Stirling's approximation:

$\log x\,! = x \log x - x.$

$\therefore$ Equation (5.23) becomes:

$$F = nE_P - 2K_BT[N \log N - (N - n) \log (N - n) - n \log n] \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.24)}$$

In thermal equilibrium at constant volume, the free energy is minimum with respect to the changes in 'n'.

$$\therefore \left(\frac{\partial F}{\partial n}\right)_T = 0 = E_P - 2K_BT \log\left(\frac{N-n}{n}\right) \quad \text{(or)} \quad \frac{E_P}{2K_BT} = \log\left(\frac{N-n}{n}\right) \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.25)}$$

Taking exponential on both sides of Equation (5.25), we get:

$$\exp\left(\frac{E_P}{2K_BT}\right) = \left(\frac{N-n}{n}\right) \quad \text{(or)} \quad n = (N-n)\exp\left(\frac{-E_P}{2K_BT}\right) \quad \text{if } n << N \text{ then:}$$

$$n \approx N \exp\left(\frac{-E_P}{2K_BT}\right) \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.26)}$$

(ii) Frenkel defects: Let the ionic crystal contains N number of atoms and the number of interstitial spaces are slightly less than the number of atoms. Let N_i be the number of interstitial spaces in a perfect crystal. The amount of energy required to displace an atom from regular atomic site to an interstitial position is E_i. At some thermal equilibrium temperature, let there be 'n' number of cation site vacancies and same number of interstitial atoms. The number of ways the 'n' Frenkel defects can be formed is:

$$P = \frac{N!}{(N-n)!\,n!} \times \frac{N_i!}{(N_i - n)!\,n!} \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.27)}$$

The increase in entropy (S) due to the creation of Frenkel defects is given by:

$$S = K_B \log P \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.28)}$$

Substituting Equation (5.27) in (5.28), we get:

$$S = K_B \log\left[\frac{N!}{(N-n)!\,n!} \times \frac{N_i!}{(N_i - n)!\,n!}\right] \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.29)}$$

These defects produce not only change in entropy but also change in free energy (F) given by:

$$F = U - TS \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.30)}$$

Equations (5.28) and (5.30) are taken from thermodynamics. Substituting Equation (5.29) in (5.30), we have:

$$F = nE_i - K_BT \log\left[\frac{N!}{(N-n)!\,n!} \times \frac{N_i!}{(N_i - n)!\,n!}\right] \quad \text{\_\_\_\_\_\_\_\_\_\_ (5.31)}$$

The logarithmic term in the above equation can be simplified by applying Stirling's approximation $\log x! = x \log x - x$.

$$\therefore \log\left[\frac{N!}{(N-n)!\,n!} \times \frac{N_i!}{(N_i-n)!\,n!}\right] = \log\frac{N!}{(N-n)!\,n!} + \log\frac{N_i!}{(N_i-n)!\,n!}$$

$$\cong N\log N + N_i \log N_i - (N-n)\log(N-n) - (N_i - n)\log(N_i - n) - 2n\log n \quad \text{——— (5.32)}$$

Substituting Equation (5.32) in (5.31), we have:

$$F = nE_i - K_B T\,[N\log N + N_i \log N_i - (N-n)\log(N-n) - (N_i - n)\log(N_i - n) - 2\,n\log n] \quad \text{——— (5.33)}$$

At thermal equilibrium, the change in free energy is minimum w.r.t. 'n', so we have:

$$\left[\frac{\partial F}{\partial n}\right]_T = 0 = E_i - K_B T \log\frac{(N-n)(N_i-n)}{n^2} \quad \text{——— (5.34)}$$

$$\therefore E_i = K_B T \log\frac{(N-n)(N_i-n)}{n^2} \qquad \text{(or)} \qquad \frac{E_i}{K_B T} = \log\frac{(N-n)(N_i-n)}{n^2}$$

Taking exponential on both sides, we get:

$$\exp\left(\frac{E_i}{K_B T}\right) = \frac{(N-n)(N_i-n)}{n^2}$$

$$n^2 = (N-n)(N_i-n)\exp\left(\frac{-E_i}{K_B T}\right) \qquad \text{if } n << N_i$$

$$n^2 \approx N\,N_i \exp\left[\frac{-E_i}{K_B T}\right] \qquad \text{(or)} \qquad n = (N\,N_i)^{1/2} \exp\left(\frac{-E_i}{2K_B T}\right) \quad \text{——— (5.35)}$$

The above equation shows that n is proportional to $(NN_i)^{1/2}$

5.5 Diffraction of X-rays by crystal planes and Bragg's law

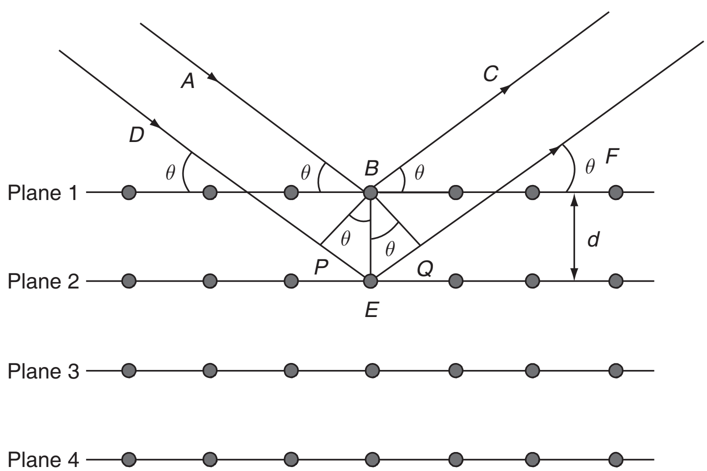
The visible light rays when pass through a sharp edge of an object can form some bright regions inside the geometrical shadow of the object. This is due to the bending nature of light, called diffraction. Diffraction of visible light rays can also be produced using plane-ruled grating. This grating consists of about 6000 lines/cm; so that the spacing between any two consecutive lines in the grating is of the order of the wavelength of visible light used to produce diffraction. The wavelength of X-rays is of the order of

an angstrom, so X-rays are unable to produce diffraction with plane optical grating. To produce diffraction with X-rays, the spacing between the consecutive lines of grating should be of the order of few angstroms. Practically, it is not possible to construct such a grating. In the year 1912, a German physicist Laue suggested that the three-dimensional arrangement of atoms in a crystal can serve as a three-dimensional grating. Inside the crystal, the spacing between the crystal planes can work as the transparent regions as between lines in a ruled grating. Laue's associates Friedrich and Knipping succeeded in diffracting X-rays by passing through a thin crystal.

In 1913, W.L. Bragg and his son W.H. Bragg gave a simple interpretation of the diffraction pattern. According to Bragg, the diffraction spots produced are due to the reflection of some of the incident X-rays by various sets of parallel crystal planes. These planes are called Bragg's planes. The Bragg's interpretation is explained in the following topic.

Bragg's law: W.L. Bragg and W.H. Bragg considered the X-ray diffraction as the process of reflection of X-rays by crystal planes as shown in Fig. 5.11. A monochromatic X-ray beam of wavelength λ is incident

Figure 5.11 Bragg's law



at an angle θ to a family of Bragg planes. Let the interplanar spacing of crystal planes is '*d*'. The dots in the planes represent positions of atoms in the crystal. Every atom in the crystal is a source of scatterer of X-rays incident on it. A part of the incident X-ray beam *AB*, incident on an atom at B in plane l, is scattered along the direction *BC*. Similarly, a part of incident X-ray *DE* [in parallel to *AB*] falls on atom at *E* in plane 2 and is scattered in the direction *EF* and it is parallel to *BC*. Let the beams *AB* and *DE* make an angle θ with the Bragg's planes. This angle θ is called the angle of diffraction or glancing angle.

If the path difference between the rays *ABC* and *DEF* is equal to λ, 2λ, 3λ… etc. or $n\lambda$, i.e., integral multiples of wavelength, where $n = 1, 2, 3,\ldots$ etc. are called first-order, second-order, third-order … etc. maxima, respectively. As path difference is equal to $n\lambda$, then the rays reflected from consecutive planes are in phase; so, constructive interference takes place among the reflected rays *BC* and *EF*, hence the resulting diffracted ray is intense. On the other hand, if the path difference between the rays *ABC* and *DEF* is $\lambda/2, 3\lambda/2, 5\lambda/2, \ldots$ etc., then the scattered rays *BC* and *EF* are out of phase so that destructive

interference takes place and hence the resulting ray intensity is minimum. To find the path difference between these rays, drop perpendiculars from B on DE and EF. The intersecting points of perpendiculars are P and Q as shown in Fig. 5.11. The path difference between the rays is $PE + QE$. From the figure, we know that BE is perpendicular to plane 1 and BP is perpendicular to AB. So, as the angle between ray AB and plane 1 is θ, then $\angle PBE = \angle QBE = \theta$. In the triangle PBE, $\sin\theta = PE/BE = PE/d$ or $PE = d \sin\theta$. Similarly, $EQ = d \sin\theta$.

$\therefore$ For constructive interference, $PE + EQ = n\lambda$ or $d\sin\theta + d\sin\theta = n\lambda$

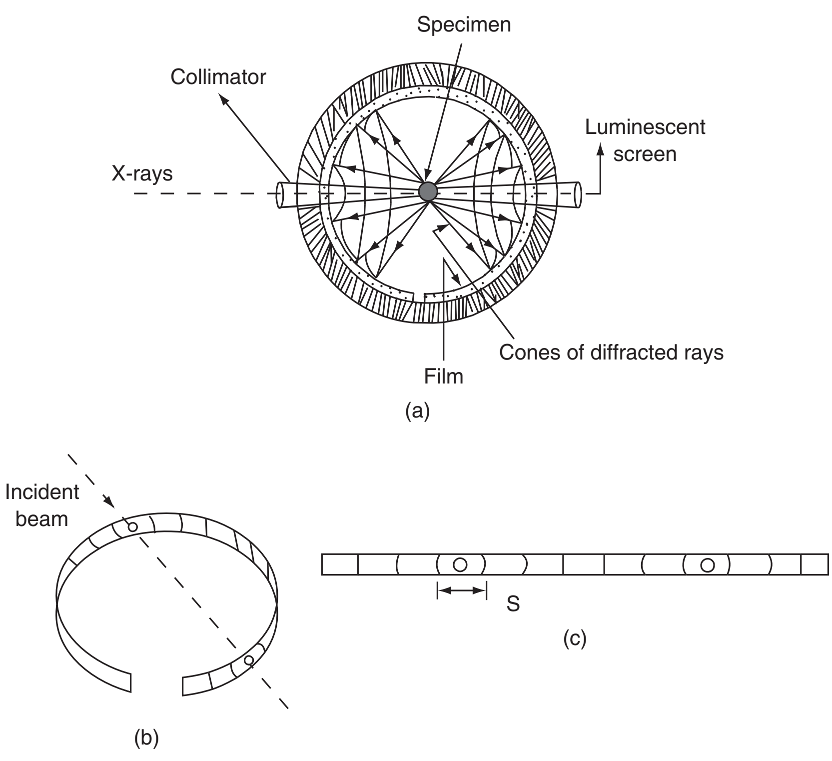
i.e., $2d\sin\theta = n\lambda$

The above equation is called Bragg's law.

5.6 Powder method

X-ray powder method is usually carried for polycrystalline materials. The powder photograph is obtained in the following way. The given polycrystalline material is ground to fine powder and this powder can be taken either in a capillary tube made up of non-diffracting material or is just struck on a hair with small quantity of binding material and fixed at the centre of cylindrical Debye-Scherrer camera as shown in Fig. 5.12(a).

Figure 5.12 (a) Debye-Scherrer cylindrical camera; (b) film mounted in camera and (c) film on stretchout



A stripe of X-ray photographic film is arranged along the inner periphery of the camera. A beam of monochromatic X-rays is passed through the collimator to obtain a narrow fine beam of X-rays. This beam falls on the polycrystalline specimen and gets diffracted. The specimen contains very large number of small crystallites oriented in random directions. So, all possible diffraction planes will be available for Bragg reflection to take place. Such reflections will take place from many sets of parallel planes lying at different angles to the incident X-ray beam. Also, each set of planes gives not only first-order reflections but also of higher orders as well. Since all orientations are equally likely, the reflected rays will form a cone whose axis lies along the direction of the incident beam and whose semi-vertical angle is equal to twice the glancing angle (θ), for that particular set of planes. For each set of planes and for each order, there will be such a cone of reflected X-rays. There intersections with a photographic film sets with its plane normal to the incident beam, form a series of concentric circular rings. In this case, a part of the reflected cone is recorded on the film and it is a pair of arcs, the resulting pattern is shown in Fig. 5.12(c). Diameter of these rings or corresponding arcs is recorded on the film, and using this the glancing angle and interplanar spacing of the crystalline substance can be determined. Figure 5.12(b) shows the film mounted in the camera and the X-ray powder pattern obtained. The film on spread-out is shown in Fig 5.12(c). The distance between any two corresponding arcs on the film is indicated by the symbol S.

In case of cylindrical camera, the diffraction angle θ is proportional to S. Then,

$\theta = \dfrac{S}{4R}$ where R represents the radius of the camera.

If S_1, S_2, S_3 … etc. are the distances between symmetrical lines on the stretched film, then,

$$\theta_1 = \frac{S_1}{4R}, \theta_2 = \frac{S_2}{4R}, \theta_3 = \frac{S_3}{4R} \ldots$$

Using these values of θ_n in Bragg's equation $n\lambda = 2\, d_{hkl} \sin \theta_n$

where $n = 1, 2, 3, \ldots$ etc = order of diffraction

d_{hkl} = interplanar spacing

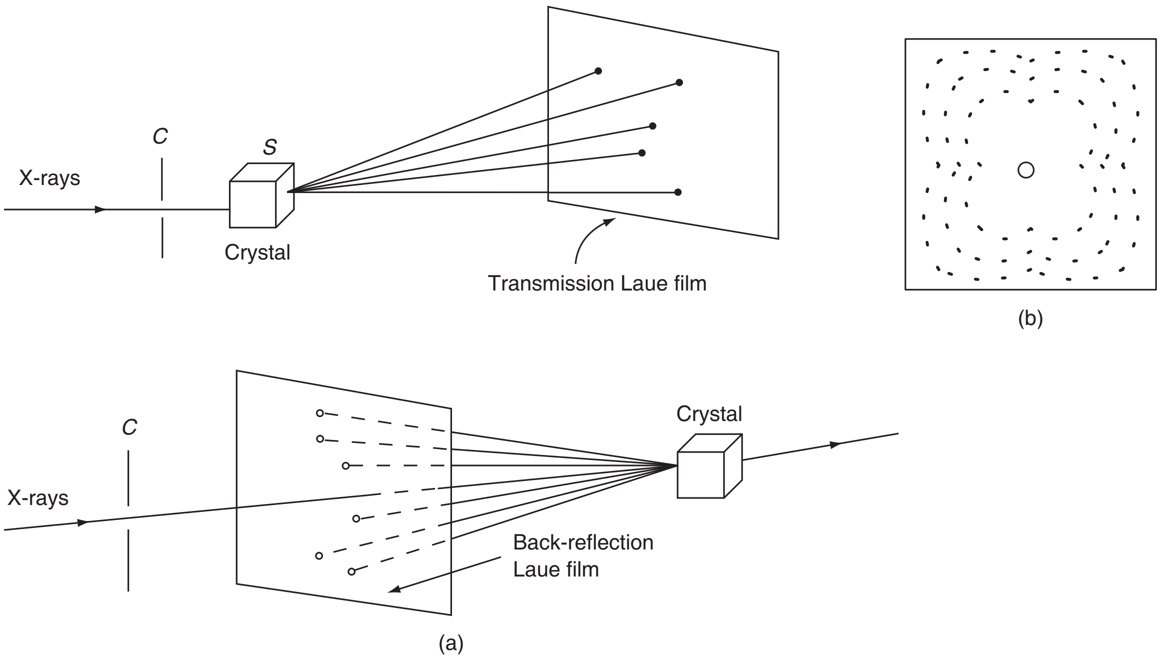
θ_n = angle of diffraction for nth order

The interplanar spacing d_{hkl} can be calculated.

5.7 Laue method

In Laue method, a narrow beam of white X-rays [usually in the wavelength range, 0.2 to 2.0 Å] is obtained by passing X-rays through a collimator 'C'. This beam is allowed to fall on a stationary single crystal 'S' as shown in Fig. 5.13(a). The crystal act as a 3-dimensional diffraction grating to the incident beam. The processes of reflection of X-rays by crystal planes is considered as X-ray diffraction. The diffraction phenomenon satisfies Bragg's law, $n\lambda = 2d \sin \theta$. where $n = 1, 2, 3, \ldots$ represent the order of diffraction, λ = wavelength of diffracted X-rays from a system of crystal planes with interplanar spacing 'd' and θ = glancing angle i.e., the angle made by X-rays with a crystal plane. As the crystal is not rotated, so, the angle 'θ' is fixed for a set of planes having separation 'd'. Different sets of crystal planes satisfy Bragg's law with different wavelengths of X-rays and produce diffraction. The diffracted X-rays from a set of planes produce constructive interference, if they are in phase and form an intense beam, and this produces dark spots on photographic film. If the diffracted rays are out of phase, they produce destructive interference so that photographic film is unaffected.

Figure 5.13 (a) X-ray diffraction by crystal plane and (b) Laue pattern for NaCl crystal



Laue photograph is obtained either by allowing the transmitted diffracted rays or by back-reflected diffracted rays on photographic film as shown in Fig. 5.13(b).

As we observe the diffracted film, the diffracted spots lie on certain curves. These curves are either ellipses or hyperbolas on transmission Laue photograph and hyperbolas on back-reflection Laue photograph. The way of arrangement of spots on a film is a characteristic property of the crystal. Laue method is useful to decide the crystal symmetry and orientation of the internal arrangement of atoms/molecules in the crystal. Cell parameters of a crystal cannot be determined using Laue method. For transmission Laue method, the crystal should be thin.

Laue method can be used to study imperfections or strains in the crystal. The presence of above defects forms streaks instead of spots in the Laue photograph.

Formulae

1. $d = \dfrac{1}{\sqrt{\dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} + \dfrac{l^2}{c^2}}}$

2. $d = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}}$

3. $n \approx N \exp\left[\dfrac{-E_V}{K_B T}\right] \leftarrow$ Metallic crystal

4. $n \approx N \exp\left[\dfrac{-E_P}{2K_B T}\right] \leftarrow$ Schottky defect

5. $n \approx (NN_i)^{1/2} \exp\left[\dfrac{-E_i}{2K_B T}\right] \leftarrow$ Frenkel defect

6. $2d \sin\theta = n\lambda$

Solved Problems

1. *A beam of X-rays of wavelength 0.071* nm *is diffracted by (110) plane of rock salt with lattice constant of 0.28* nm. *Find the glancing angle for the second-order diffraction.*
(Set-1–Sept. 2007), (Set-2, Set-3–Sept. 2006), (Set-2–May 2006), (Set-3–May 2004), (Set-4–May 2003)

Sol: Given data are:

Wavelength (λ) of X-rays = 0.071 nm

Lattice constant (a) = 0.28 nm

Plane (hkl) = (110)

Order of diffraction = 2

Glancing angle θ = ?

Bragg's law is $2d \sin\theta = n\lambda$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}, \text{ because rock salt is FCC}$$

$$= \frac{0.28 \times 10^{-9}}{\sqrt{1^2 + 1^2 + 0^2}}\,\text{m} = \frac{0.28 \times 10^{-9}}{\sqrt{2}}\,\text{m}$$

Substitute in Bragg's equation

$$2 \times \frac{0.28 \times 10^{-9}}{\sqrt{2}} \sin\theta = 2 \times 0.071 \times 10^{-9}$$

$$\sin\theta = \sqrt{2} \times \frac{0.071}{0.28} = 0.3586$$

$$\theta = \sin^{-1}(0.3586) = 21.01° \approx 21°$$

2. *A beam of X-rays is incident on a NaCl crystal with lattice plane spacing 0.282* nm. *Calculate the wavelength of X-rays if the first-order Bragg reflection takes place at a glancing angle of 8°35′. Also calculate the maximum order of diffraction possible.*
(Set-4–Sept. 2007), (Set-3–May 2007), (Set-2–May 2004), (Set-3–May 2003)

Sol: Given data are:

NaCl crystal is FCC

Lattice plane spacing (d) = 0.282 nm

Wavelength of rays (λ) = ?

Order of diffraction (n) = 1

Glancing angle $\theta = 8°35'$

Bragg's equation is $n\lambda = 2d \sin\theta$

$$1\lambda = 2 \times 0.282 \times 10^{-9} \sin(8°35')$$

$$= 0.0842 \text{ nm}$$

Maximum order of diffraction (n_{max}) = ?

$2d \sin\theta = n\lambda$

if $\theta = 90°$ then $n = n_{max}$

$\therefore\ 2d = n_{max}\,\lambda$

$$n_{max} = \frac{2d}{\lambda} = \frac{2\times 0.282\text{ nm}}{0.0842\text{ nm}} = 6.7 \approx 7$$

3. *The fraction of vacant sites in a metal is 1 × 10^−10 at 500°* C. *What will be the fraction of vacancy sites at 1000°* C?

(Set-4–Sept. 2006), (Set-1–May 2004), (Set-2–May 2003)

Sol: The number of vacancies at temperature (TK) in a metal is represented by:

$$n \approx N \exp\left(\frac{-E_V}{K_B T}\right) \qquad \text{(or)} \qquad \frac{n}{N} = \exp\left(\frac{-E_V}{K_B T}\right)$$

The given data are:

$$\frac{n}{N} = 1\times 10^{-10} \text{ at } 500°\text{ C} \quad \text{or} \quad 773\text{ K} \qquad \frac{n'}{N} = ?\text{ at } 1000°\text{ C} \quad \text{or} \quad 1273\text{ K}$$

$$1\times 10^{-10} = \exp\left(\frac{-E_V}{K_B 773}\right) \quad \underline{\qquad\qquad} (1)$$

$$\frac{n'}{N} = \exp\left(\frac{-E_V}{K_B\, 1273}\right) \quad \underline{\qquad\qquad} (2)$$

Taking logarithms on both sides of the above Equations (1) and (2), we get:

$$\ln 10^{-10} = \frac{-E_V}{773\, K_B} \quad \underline{\qquad\qquad} (3)$$

$$\ln\left(\frac{n'}{N}\right) = \frac{-E_V}{1273\, K_B} \quad \underline{\qquad\qquad} (4)$$

Dividing Equation (4) by (3),

$$\frac{\ln\left(\frac{n'}{N}\right)}{\ln 10^{-10}} = \frac{\left(\frac{-E_V}{1273\, K_B}\right)}{\frac{-E_V}{773\, K_B}} = \frac{773}{1273} = 0.60723$$

$$= \ln\left(\frac{n'}{N}\right) = \ln 10^{-10} \times 0.60723$$

$$= -23.026 \times 0.60723 = -13.982$$

Take exponential on both sides,

$$\frac{n'}{N} = \exp[-13.982]\times 10^{-7} = 8.466\times 10^{-7}$$

The fraction of vacancy sites at 1000°C is 8.466×10^{-7}.

4. *Calculate the ratios d_{100}: d_{110}: d_{111} for a simple cubic structure.*

(Set-2–Nov. 2004), (Set-2–Nov. 2003)

Sol: Let 'a' be the lattice constant of cubic structure, then,

$$d_{100} = \frac{a}{\sqrt{1^2+0^2+0^2}} = a$$

$$d_{110} = \frac{a}{\sqrt{1^2+1^2+0^2}} = \frac{a}{\sqrt{2}}$$

$$d_{111} = \frac{a}{\sqrt{1^2+1^2+1^2}} = \frac{a}{\sqrt{3}}$$

The ratios $d_{100} : d_{110} : d_{111} = a : \frac{a}{\sqrt{2}} : \frac{a}{\sqrt{3}}$

$$= 1 : \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{3}} = \sqrt{6} : \sqrt{3} : \sqrt{2}$$

5. *The Bragg's angle in the first order for (220) reflection from nickel (FCC) is 38.2°. When X-rays of wavelength 1.54 Å are employed in a diffraction experiment. Determine the lattice parameter of nickel.*

(Set-2–May 2008)

Sol: Order of diffraction, $n = 1$

Diffraction angle, $\theta = 38.2°$

Wavelength of light, $\lambda = 1.54$ Å

Plane of reflection = (220)

Lattice parameter, $a = ?$

Bragg's law is $2d \sin\theta = n\lambda$

$$d = \frac{n\lambda}{2\sin\theta} = \frac{1\times 1.54}{2\times \sin 38.2} \text{ Å}$$

Also $$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

$$a = d\times\sqrt{h^2+k^2+l^2}$$

$$= \frac{1\times 1.54}{2\times \sin 38.2}\times\sqrt{2^2+2^2+0^2}$$

$$= \frac{4.35578}{1.23682} = 3.522$$

6. *Monochromatic X-rays of $\lambda = 1.5$ A.U are incident on a crystal face having an interplanar spacing of 1.6 A.U. Find the highest order for which Bragg's reflection maximum can be seen.*

(Set-4–May 2006)

Sol: Given data are

Wavelength of light (λ) = 1.5 Å

Interplanar spacing (d) = 1.6 Å

Glancing angle (θ_m) = 90°

Order of diffraction (n) = ?

Bragg's law

$n\lambda = 2d \sin\theta$

$$n = \frac{2d\sin\theta}{\lambda} = \frac{2 \times 1.6 \times \sin 90}{1.5} = \frac{3.2}{1.5}$$

$= 2.13 \approx 2$

$\therefore$ The maximum order of diffraction is 2

7. *The distance between (110) planes in a body centred cubic structure is 0.203 nm. What is the size of the unit cell? What is the radius of the atom?*

(Set-3–Sept. 2007), (Set-3–May 2006)

Sol: The given data are

The distance between (110) planes of BCC structure (d_{110}) = 0.203 nm = 0.203×10^{-9} m

Length of unit cell (a) = ?

Volume of unit cell (a^3) = ?

Radius of the atom (r) = ?

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$0.203 \times 10^{-9} = \frac{a}{\sqrt{1^2 + 1^2 + 0^2}} = \frac{a}{\sqrt{2}}$$

$$a = 0.203 \times \sqrt{2} = 0.287 \times 10^{-9}\,\text{m}$$

Volume of unit cell $a^3 = 0.02364 \times 10^{-27}\,\text{m}^3$

$$\text{Radius of atom } (r) = \frac{\sqrt{3}a}{4} = \frac{1.732 \times 0.287 \times 10^{-9}}{4}$$

$= 0.1243 \times 10^{-9}$ m

8. *Monochromatic X-rays of $\lambda = 1.5$ A.U. are incident on a crystal face having an interplanar spacing of 1.6 A.U. Find the highest order for which Bragg's reflection maximum can be seen.*

(Set-1–Sept. 2006)

Sol: Given data are wavelength of X-rays, $\lambda = 1.5$ Å

Interplanar spacing, $d = 1.6$ Å

For highest order of diffraction, $\theta = 90°$

Highest order of diffraction, n = ?

Formula $2d \sin \theta = n\,\lambda$

$2 \times 1.6 \times \sin 90^\circ = n \times 1.5$

$$n = \frac{2 \times 1.6}{1.5} = 2.13 \approx 2$$

$\therefore$ Highest order of diffraction is 2.

9. *Calculate the glancing angle at (110) plane of a cubic crystal having axial length 0.26* nm *corresponding to the second order diffraction maximum for the X-rays of wavelength 0.65* nm.

(Set-1–May 2007)

Sol: The given data are

Edge length of cubic system, $a = 0.26$ nm

Wavelength of X-rays $\lambda = 0.065$ nm

Glancing angle, for plane (110), $\theta = ?$

Order of diffraction, $n = 2$

Separation between (110) planes of a cube, $d = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}} = \dfrac{0.26}{\sqrt{1^2 + 1^2 + 0^2}}$ nm

$$= \frac{0.26}{\sqrt{2}} = 0.184 \text{ nm}$$

Bragg's law

$$2d \sin \theta = n\lambda$$

$$2 \times 0.184 \text{ nm} \times \sin \theta = 2 \times 0.065 \text{ nm}$$

$$\sin \theta = \frac{0.065}{0.184} = 0.353$$

$$\therefore \theta = \sin^{-1}(0.353)$$

$$= 20^\circ 41' 13''$$

10. *The Bragg's angle for reflection from the (111) plane in a FCC crystal is 19.2° for an X-ray wavelength of 1.54* A.U. *Compute the cube edge of the unit cell.*

(Set-2, Set-4–May 2007)

Sol: The given data are

Bragg's angle, $\theta = 19.2^\circ$

Wavelength of X-rays, $\lambda = 1.54$ Å

Order of diffraction, $n = 1$

Cube edge, $a = ?$

Bragg's law

$2d \sin \theta = n\lambda$

$2d \sin 19.2^\circ = 1 \times 1.54$

$$d = \frac{1.54}{2 \times \sin 19.2^\circ} = \frac{1.54}{2 \times 0.3289}$$

$$= 2.3411 \text{ Å}$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

or $$a = d\sqrt{h^2 + k^2 + l^2}$$
$$= 2.3411 \times \sqrt{1^2 + 1^2 + 1^2}$$
$$= 2.3411 \times \sqrt{3} = 4.05\,\text{Å}$$

11. *The Bragg's angle in the first order for (220) reflection from nickel (FCC) is 38.2°. When X-rays of wavelength 1.54* Å *are employed in a diffraction experiment. Determine the lattice parameter of nickel.*

(Set-2–May 2008)

Sol: Order of diffraction, $n = 1$

Diffraction angle , $\theta = 38.2°$

Wavelength of light, $\lambda = 1.54$ Å

Plane of reflection = (220)

Lattice parameter, a = ?

Bragg's law is 2d sin$\theta = n\lambda$

$$d = \frac{n\lambda}{2\sin\theta} = \frac{1 \times 1.54}{2 \times \sin 38.2°}\,\text{Å}$$

Also $$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$a = d \times \sqrt{h^2 + k^2 + l^2}$$

$$= \frac{1 \times 1.54}{2 \times \sin 38.2°} \times \sqrt{2^2 + 2^2 + 0^2}$$

$$= \frac{4.35578}{1.23682} = 3.522\text{ Å}$$

12. *Copper has FCC structure with lattice constant 0.36* nm. *Calculate the interplanar spacing for (111) and (321) planes.*

Sol: Given data is:

lattice constant (a) = 0.36 nm = 0.36×10^{-9} m

Interplanar spacing (d) for (111) plane is:

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{0.36 \times 10^{-9}}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{0.36 \times 10^{-9}}{\sqrt{3}}\text{ m}$$

= 0.208×10^{-9} m = 0.208 nm

Interplanar spacing for (321) plane

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{0.36 \times 10^{-9}}{\sqrt{3^2 + 2^2 + 1^2}} = \frac{0.36 \times 10^{-9}}{\sqrt{9 + 4 + 1}} = \frac{0.36 \times 10^{-9}}{\sqrt{14}}\text{ m}$$

= 0.096×10^{-9} m

= 0.096 nm.

13. *The first-order diffraction occurs when a X-ray beam of wavelength 0.675 Å incident at a glancing angle of 5°25′ on a crystal. What is the glancing angle for third-order diffraction to occur?*

Sol: Wavelength of X-rays (λ) = 0.675 Å

Glancing angle for first order ($n = 1$) diffraction (θ_1) = 5°25′

Find the glancing angle for third order ($n = 3$) diffraction (θ_3) = ?

Bragg's equation is $2d \sin\theta = n\lambda$

For first order, $2d \sin\theta_1 = 1\lambda$

$2d \sin 5°25' = 0.675 \times 10^{-10}$ m

$$d = \frac{0.675 \times 10^{-10}}{2 \sin 5°25'} \text{ m} = \frac{0.675 \times 10^{-10}}{0.1888} = 3.575 \times 10^{-10} \text{ m} = 3.575 \text{ Å}$$

For third-order diffraction,

$2d \sin\theta_3 = 3\lambda$

$$\sin\theta_3 = \frac{3\lambda}{2d} = \frac{3 \times 0.675 \times 10^{-10}}{2 \times 3.575 \times 10^{-10}} = 0.283$$

$$\theta_3 = \sin^{-1}(0.283) = 16.45° = 16°28'.$$

14. *What is the angle at which the third-order reflection of X-rays of 0.79 Å wavelength can occur in a calcite crystal of 3.04×10^{-8} cm spacing?*

Sol: Wavelength of X-rays, $\lambda = 0.79$ Å $= 0.79 \times 10^{-8}$ cm

Interplanar spacing, $d = 3.04 \times 10^{-8}$ cm

Order of diffraction, $n = 3$

Angle of diffraction, θ = ?

$2d \sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{2d} = \frac{3 \times 0.79 \times 10^{-8}}{2 \times 3.04 \times 10^{-8}} = 0.3898$$

$$\theta = \sin^{-1}(0.3898)$$

$$= 25°\, 29'\, 28''$$

Multiple-choice Questions

1. Crystal directions are defined as ___________ .

(a) certain directions inside the crystal along which large concentration of atoms exists
(b) certain directions inside the crystal along which low concentration of atoms exists
(c) certain directions inside the crystal along which no atoms are present
(d) none

2. Crystal planes and directions can be represented by a set of __________ small integers.

   (a) 2 (b) 3 (c) 4 (d) 6

3. To represent crystal direction, the Miller indices should be enclosed in __________

   (a) square brackets (b) round brackets
   (c) curly brackets (d) none

4. If the Miller indices of two planes are (211) and (422), then they are __________ .

   (a) parallel (b) perpendicular
   (c) they are at an angle of 45° (d) none

5. If the Miller indices of a plane along Y and Z-direction is zero, then __________ .

   (a) the plane is perpendicular to X-axis (b) the plane is parallel to Y-axis
   (c) the plane is parallel to X-axis (d) the plane is parallel to Z-axis

6. If the Miller indices of a plane is $\left(h\bar{k}l\right)$, then the plane __________ .

   (a) intersects negative X-axis (b) intersects negative Z-axis
   (c) intersects negative Y-axis (d) intersects positive Y-axis

7. If {*hkl*} are the Miller indices in cubic system, they represent __________ .

   (a) (100) and $(\bar{1}00)$ planes (b) (010) and $(0\bar{1}0)$ planes
   (c) (001) and $(00\bar{1})$ planes (d) all

8. The Miller indices <*hkl*> in cubic system represent the following directions __________ .

   (a) $\left[\bar{1}11\right], \left[1\bar{1}1\right]$ and $\left[11\bar{1}\right]$ (b) $\left[1\bar{1}\bar{1}\right], \left[\bar{1}1\bar{1}\right]$ and $\left[\bar{1}\bar{1}1\right]$
   (c) $\left[111\right]$ and $\left[\bar{1}\bar{1}\bar{1}\right]$ (d) all

9. If (*hkl*) represents the Miller indices of planes in cubic crystal of lattice constant '*a*', the separation between the parallel planes is __________ .

   (a) $\dfrac{a}{h^2+k^2+l^2}$ (b) $\dfrac{a}{h+k+l}$
   (c) $\dfrac{a}{\sqrt{h^2+k^2+l^2}}$ (d) $\dfrac{a}{\sqrt{h+k+l}}$

10. Crystal defects are __________ .

    (a) point and line defects (b) surface defects
    (c) volume defects (d) all

11. Point defects are __________ .

    (a) lattice site defects (b) compositional defects
    (c) electronic defects (d) all

12. Electrical charge neutrality is maintained in __________ .

    (a) Schottky defect (b) Frenkel defect
    (c) both a and b (d) none

13. Schottky defect may exist in __________ .

(a) NaCl crystal (b) KCl crystal
(c) KBr crystal (d) all

14. Substitutional defect and interstitial impurity defect belong to __________ .

(a) compositional defect (b) Schottky defect
(c) Frenkel defect (d) lattice site defects

15. Non-uniformity of charge or energy distribution in the crystal is referred to as __________ .

(a) point defect (b) electronic defect
(c) Schottky defect (d) Frenkel defect

16. Point defects in crystals are formed by __________ .

(a) thermal fluctuations (b) large deformation
(c) bombarding with high energetic particles (d) all

17. Edge dislocation and screw dislocation belong to __________ .

(a) electronic defects (b) compositional defects
(c) line defects (d) point defects

18. Just above the edge of an incomplete crystal plane in a crystal, the bond distances are __________ .

(a) equal to normal values (b) lesser than normal values
(c) greater than normal values (d) none

19. If the incomplete plane is below the slip plane, then the edge dislocation is __________ .

(a) positive (b) negative
(c) both a and b (d) none

20. In edge dislocation, the Burger's vector is __________ to the dislocation line.

(a) parallel (b) at an angle of 45°
(c) perpendicular (d) at an angle of 60°

21. If E_V is the energy required to form a vacancy in the crystal containing 'N' atoms at temperature 'T', the number of vacancies in the crystal is __________ [K_B = Boltzmann constant].

(a) $N \exp\left[\frac{-E_V}{K_B T}\right]$ (b) $N \exp\left[\frac{-K_B T}{E_V}\right]$

(c) $N \exp\left[\frac{E_V}{K_B T}\right]$ (d) $N \exp\left[\frac{-E_V}{2K_B T}\right]$

22. If E_P is the energy required to move an anion–cation pair from interior to the surface of an ionic crystal containing N pairs of ions, the formation of n pairs of vacancies at temperature 'T' is given by __________ [K_B = Boltzmann constant].

(a) $N \exp\left[\frac{-E_P}{K_B T}\right]$ (b) $N \exp\left[\frac{E_P}{2K_B T}\right]$

(c) $N \exp\left[\frac{-E_P}{2K_B T}\right]$ (d) $N \exp\left[\frac{E_P}{K_B T}\right]$

23. Let E_i is the energy required to move a cation [positive ion] to the interstitial space of an ionic crystal containing 'N' number of atoms and N_i be the number of interstitial spaces. The number of ways n cations are moved to interstitial spaces, at temperature T is given by __________ [K_B = Boltzmann constant].

(a) $(NN_i)^{1/2} \exp\left(\frac{-E_i}{2K_BT}\right)$ (b) $(NN_i)^{1/2} \exp\left(\frac{E_i}{2K_BT}\right)$

(c) $(NN_i)^{1/2} \exp\left(\frac{-E_i}{K_BT}\right)$ (d) $(NN_i) \exp\left(\frac{-E_i}{2K_BT}\right)$

24. If a monochromatic X-ray of wavelength 'λ' incident at an angle 'θ' on a parallel set of crystal planes of separation 'd', then the Bragg's law for constructive interference is __________ [$n = 1, 2, 3, \ldots$ = order of diffraction].

(a) $2d \sin\theta = n\lambda$ (b) $d \sin\theta = n\lambda$
(c) $2\lambda \sin\theta = nd$ (d) $\lambda \sin\theta = nd$

25. Miller indices of the plane parallel to X and Y axes are __________ .

(a) (001) (b) (010) (c) (100) (d) (111)

26. The crystal planes are defined as some imaginary planes inside a crystal in which __________ of atoms are present.

(a) large concentration (b) low concentration
(c) medium concentration (d) none

27. Crystal planes and directions can be represented by a set of three small integers called __________ .

(a) plane indices (b) Miller indices (c) direction indices (d) none

28. If the Miller indices are enclosed in round brackets, then it represents a crystal __________ .

(a) plane (b) direction (c) set of directions (d) system of planes

29. Miller indices may be defined as a set of three integers obtained by clearing the reciprocals of the __________ made by a plane on crystallographic axes.

(a) intercepts (b) relations (c) both a and b (d) none

30. Miller indices represent a set of equidistant __________ planes.

(a) perpendicular (b) intersecting
(c) parallel (d) none

31. If (hkl) is the Miller indices of a plane, then the plane divides the lattice constant 'a' along 'X' axis into __________ .

(a) h equal parts (b) k equal parts (c) l equal parts (d) all

32. Point defects in crystals are also called as __________ defects.

(a) three-dimensional (b) two-dimensional
(c) one-dimensional (d) zero-dimensional

33. By moving an anion and a cation from interior of an ionic crystal to the surface of the crystal leads to __________ defect.

(a) Frenkel (b) Schottky (c) both a and b (d) none

34. Vacancies and interstitial defects belong to __________ defects.

(a) lattice site (b) Schottky (c) Frenkel (d) none

35. The crystal defect formed by moving a cation to interstitial spaces in an ionic crystal is known as ___________ defect.

    (a) Schottky (b) point defect (c) Frenkel (d) none

36. Examples for Frenkel defect are ___________ .

    (a) CaF_2 (b) AgBr (c) AgI (d) all

37. Presence of impure atoms in the crystal leads to ___________ defects.

    (a) Schottky (b) Frenkel (c) compositional (d) none

38. If '*r*' is the radius of a parent atom of a crystal, then octahedral and tetrahedral spaces can accommodate an interstitial atom of radius ___________ and ___________ , respectively.

    (a) $0.414r$, $0.225r$ (b) $0.225r$, $0.414r$
    (c) $0.0225r$, $0.0414r$ (d) $0.0414r$, $0.0225r$

39. Extrinsic semiconductors contain ___________ crystal defect.

    (a) interstitial (b) substitutional (c) Frenkel (d) Schottky

40. Just below the edge of an incomplete crystal plane in a crystal, the bond distances are ___________ normal values.

    (a) same as (b) more than (c) less than (d) none

41. If a crystal plane ends some where inside the crystal, then the defect along the edge of the incomplete plane is called ___________ .

    (a) edge dislocation (b) screw dislocation
    (c) Schottky defect (d) interstitial defect

42. If the incomplete plane is above the slip plane in the crystal, then the edge dislocation is ___________ .

    (a) Schottky (b) Frenkel (c) negative (d) positive

43. The magnitude and direction of the displacement of crystal planes due to edge dislocation can be represented by a vector called ___________ .

    (a) Burger's vector (b) Laue's vector (c) both a and b (d) none

44. In screw dislocation, the atoms at one end of a plane are displaced by ___________ distance with respect to the other end of the plane, perpendicular to plane.

    (a) 3 atomic (b) 2 atomic (c) 1 atomic (d) none

45. In screw dislocation, Burger's vector is ___________ to dislocation line.

    (a) perpendicular (b) parallel (c) both a and b (d) none

46. By decreasing the equilibrium temperature of a crystal, the concentration of vacancies ___________ .

    (a) decreases (b) increases (c) remains the same (d) none

47. To produce diffraction with X-rays, the spacing between the consecutive lines of grating should be of the order of ___________ angstroms.

    (a) thousands of (b) hundreds of (c) few (d) none

48. In 1912, Laue suggested that a crystal can serve as a ___________ for X-ray diffraction.

    (a) three-dimensional grating (b) two-dimensional grating
    (c) one-dimensional grating (d) none of the above

49. ___________ and ___________ succeeded in diffracting X-rays by passing through a thin crystal.

(a) Friedrich and Knipping (b) Bragg and Knipping
(c) Friedrich and Laue (d) none of the above

50. If the path difference between the X-rays reflected by successive crystal planes is $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$, then the intensity of diffracted ray ___________ .

(a) will not change (b) is minimum (c) is maximum (d) none

51. If the path difference between the X-rays reflected by successive crystal planes is $n\lambda$, where $n = 1, 2, 3, \ldots$, then the intensity of diffracted ray ___________ .

(a) is minimum (b) is maximum (c) remains the same (d) none

52. X-ray powder method is usually carried for ___________ materials.

(a) polycrystalline (b) powder (c) single crystal (d) amorphous

53. Using powder diffraction, ___________ of a crystal can be determined.

(a) the interatomic spacing (b) the interplanar spacing
(c) both a and b (d) none

54. In powder method, ___________ chromatic X-rays are used.

(a) mono (b) poly (c) both a and b (d) none

55. In Laue method, ___________ X-rays are used.

(a) monochromatic (b) white (c) both a and b (d) none

56. In transmission Laue method, the diffracted spots lie on the curves of ___________ .

(a) ellipses (b) hyperbolas (c) a or b (d) none

57. In back reflection Laue method, the diffracted spots lie on curves of ___________ .

(a) hyperbola (b) parabolas (c) ellipses (d) none

58. Laue method is useful to decide the ___________ and orientation of the internal arrangement of atoms/molecules in the crystal.

(a) cell parameters (b) crystal symmetry (c) both a and b (d) none

59. The diffracted spots will be in the form of ___________ , if the crystal contains imperfections or strains.

(a) streaks (b) spots (c) both a and b (d) none

Answers

1. a	2. b	3. a	4. a	5. a	6. c	7. d	8. d	9. c	10. d	11. d
12. c	13. d	14. a	15. b	16. d	17. c	18. b	19. b	20. c	21. a	22. c
23. a	24. a	25. a	26. a	27. b	28. a	29. a	30. c	31. a	32. d	33. b
34. a	35. c	36. d	37. c	38. a	39. b	40. b	41. a	42. d	43. a	44. c
45. b	46. a	47. c	48. a	49. a	50. b	51. b	52. a	53. b	54. a	55. b
56. c	57. a	58. b	59. a							

Review Questions

1. Derive Bragg's law of X-ray diffraction.
(Set-1–Sept. 2006), (Set-4–May 2006), (Set-3–May 2003), (Set-3–Nov. 2003)
2. What are Miller indices? How are they obtained?
(Set-1–May 2006), (Set-1, Set-2, Set-3, Set-4–June 2005), (Set-4–Nov. 2004), (Set-1–May 2003), (Set-4–Nov. 2003)
3. Explain Schottky and Frenkel defects with the help of suitable figures.
(Set-2–Sept. 2007), (Set-2–May 2007), (Set-4–Sept. 2006), (Set-1, Set-2, Set-3, Set-4–June 2005), (Set-4–Nov. 2004), (Set-1–May 2003), (Set-4–Nov. 2003)
4. State and explain Bragg's law.
(Set-1–Sept. 2007), (Set-2, Set-3–Sept. 2006), (Set-2–May 2006), (Set-3–May 2004), (Set-4–May 2003)
5. Describe with a suitable diagram, the powder method for the determination of crystal structure.
(Set-1–Sept. 2007), (Set-2–Sept. 2006), (Set-2, Set-3–May 2006), (Set-3–May 2004), (Set-4–May 2003)
6. Explain Bragg's law of X-ray diffraction.
(Set-2, Set-4–Sept. 2007), (Set-2, Set-3, Set-4–May 2007), (Set-1–May 2006), (Set-4–Sept. 2006), (Set-3–Nov. 2004), (Set-2–May 2004)
7. Describe Laue's method for determination of crystal structure.
(Set-2–May 2008), (Set-2, Set-4–Sept. 2007), (Set-3–May 2007), (Set-4–Sept. 2006), (Set-2–May 2004), (Set-3–May 2003)
8. Explain the significance of Miller indices. (Set-1–May 2004), (Set-2–May 2003)
9. Derive an expression for the number of Schottky defects in equilibrium at a temperature T.
(Set-4–Sept. 2006), (Set-1–May 2004), (Set-2–May 2003)
10. Explain the various point defects in a crystal. (Set-1–Sept. 2007), (Set-1–Nov. 2004), (Set-1–Nov. 2003)
11. Obtain the expression for the equilibrium concentration of vacancies in a solid at a given temperature.
(Set-1–Sept. 2007), (Set-1–Nov. 2004), (Set-1–Nov. 2003)
12. Deduce the expression for the interplanar distance in terms of Miller indices for a cubic structure.
(Set-3–Sept. 2008), (Set-2–Nov. 2004), (Set-2–Nov. 2003)
13. Sketch the following planes of a cubic unit cell: (001), (120) and $(\bar{2}11)$.
(Set-2–Sept. 2007), (Set-2–Nov. 2004), (Set-2–Nov. 2003)
14. Define Miller indices. Sketch the following atomic planes in a simple cubic structure (010), (110) and (111).
(Set-4–May 2004)
15. How can the interplanar spacing of a set of Miller planes be calculated in terms of Lattice parameters?
(Set-4–May 2004)
16. What is Bragg's law? Explain. (Set-2–May 2008)
17. What are Miller Indices? Draw (111) and (110) planes in a cubic lattice. (Set-2, Set-4–May 2007)
18. Draw the (112) and (120) planes and the [112] and [120] directions of a simple cubic crystal.
(Set-1–May 2007)
19. Sketch the following planes of a cubic unit cell: (001), (120) and $(\bar{2}11)$. (Set-4–Sept. 2006)
20. What is Frenkel defect? Explain. (Set-1, Set-3–May 2007)

21. Describe edge and screw dislocations. Draw Burger's circuit and slip planes for them.
(Set-4–Sept. 2007), (Set-4–May 2006)

22. Explain the significance to Burger's vector.
(Set-2, Set-4–Sept. 2007), (Set-2–May 2007), (Set-3–Sept. 2006), (Set-4–May 2006)

23. Describe Bragg's X-ray spectrometer and explain how Bragg's law can be verified.
(Set-1–Sept. 2006), (Set-4–May 2006)

24. Explain the influence of point defects in crystals and how do they affect the properties of materials.
(Set-3–Sept. 2007)

25. Obtain an expression for the energy required to create a vacancy in the crystal. (Set-3–Sept. 2007)

26. Derive an expression for the interplanar spacing in the case of a cubic structure? (Set-1–May 2007)

27. Derive an expression for the energy change due to creation of vacancies inside a solid. (Set-2–May 2006)

28. Derive an expression for the concentration of Frenkel defects present in a crystal at any temperature.
(Set-1, Set-3–May 2007)

29. Sketch the planes (120), $(2\bar{1}3)$ and directions [100] and [211]. (Set-4–Sept. 2008)

30. Explain how the X-ray diffraction can be employed to determine the crystal structure. Give the ratio of interplanar distances of (100), (110) and (111) planes for a simple cubic structure.
(Set-3–Sept. 2007), (Set-3–May 2006)

31. Distinguish between Frenkel defects and Schottky defects. (Set-2–May 2006)

32. Explain edge dislocation, screw dislocation and significance of Burger's vector. (Set-3–Sept. 2006)

33. Write short notes on Burger's vector in dislocations.

34. What are Miller indices? Derive an expression for the interplanar spacing between two adjacent planes of Miller indices (*hkl*) in a cubic lattice of edge length '*a*'.

35 Explain and illustrate, with neat sketches, the edge and screw dislocations; show the Burger's vector in them.

36. Write short notes on interstitial defects of crystals.

37. What are point defects in crystals? Derive an expression for the concentration of Schottky defect in a crystal.

38. Explain the principle, procedure and advantage of Debye–Scherrer method of X-ray diffraction.

39. Mention the different kinds of crystal imperfections.

40. Compare and contrast Frenkel and Schottky defects.

41. Write short notes on screw dislocation.

42. What are crystal imperfections? Explain.

43. Distingu ish between edge and screw dislocations. What is Burger's vector?

44. Discuss the Schottky defect in the case of ionic crystals.

45. Explain the powder method of crystal structure analysis.

46. What are Miller indices? How they are determined?

47. Show that the number of Frenkel defects in equilibrium at a given temperature is proportional to $(NN_i)^{1/2}$, where N be the number of atoms and N_i be the number of interstitial atoms.

48. Obtain the Miller indices of a plane which intercepts at a, $b/2$ and $3c$ in simple cubic unit cell. Draw a neat diagram showing the plane.

49. What do you understand by Miller indices of a crystal plane? Show that in a cubic crystal the spacing between consecutive parallel planes of Miller indices (hkl) is given by:

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

50. Define Schottky defect and derive an expression for the density of Schottky defects at a specified temperature.
51. Calculate the first nearest neighbour atom distance in ZnS (i.e., from Zn to S atoms) system.
52. Derive an expression for the interplanar distance in the case of cubic systems following Miller indices concept.
53. Define a Frenkel defect and derive an expression for the density of such defects as a function of temperature.
54. Write short notes on the Burger's vector in dislocation with appropriate diagrams.
55. Derive the Bragg's law of X-ray diffraction and obtain the relation that connects the interplanar distance 'd' in orthogonal systems with lattice parameters a, b and c.

CHAPTER 6

Lasers

6.1 Introduction

LASER stands for Light Amplification by Stimulated Emission of Radiation. Laser light is different from conventional light. In conventional light sources [such as tube light or electric bulb], there is no coordination among different atoms emitting radiation. Whereas in lasers, all atoms act together and produce highly directional, monochromatic, coherent and stimulated radiation. In conventional light source, different atoms emit radiation at different times and in different directions so that there is no phase relation between the emitted photons. The photons emitted by different atoms of laser are in phase or they maintain constant phase relationship and they move in the same direction. Lasing has been extended upto γ-rays. γ-ray lasers are called Grasers.

6.2 Characteristics of laser radiation

Laser radiation has the following important characteristics over ordinary light source. They are: i) monochromaticity, ii) directionality, iii) coherence and iv) brightness.

(i) Monochromaticity: A laser beam is more or less in single wavelength. i.e., the line width of laser beams are extremely narrow. The wavelengths spread of conventional light sources is usually 1 in 10^6, whereas in case of laser light it will be 1 in 10^{15}. i.e., if the frequency of radiation is 10^{15} Hz, then the width of line will be 1 Hz. So, laser radiation is said to be highly monochromatic. The degree of non-monochromaticity has been expressed as $\xi = d\lambda/\lambda = d\nu/\nu$ where dλ or dν is the variation in wavelength or variation in frequency of radiation.

(ii) Directionality: Laser beam is highly directional because laser emit light only in one direction. It can travel very long distances without divergence. So, laser communication has been carried between earth and moon.

A laser beam sent from earth to moon was recorded on earth after reflection by moon. The directionality of a laser beam has been expressed in terms of divergence. Suppose r_1 and r_2 are the radii of laser beam at distances D_1 and D_2 from a laser, then we have:

$$\text{The divergence, } \Delta\theta = \frac{r_2 - r_1}{D_2 - D_1}$$

The divergence for a laser beam is 0.01 milliradian whereas in case of search light it is 0.5 radian.

(iii) Coherence: Laser beam is spatially and temporally coherent.

Spatial coherence: If a wave maintains a constant phase difference or in phase at two different points on the wave over a time 't', then the wave is said to have spatial coherence. For He-Ne gas laser, the coherence length 'L_c' is about 600 Km. Coherence length is defined as the length over which the wave maintains same phase. For sodium lamp light source, the coherent length is 3 cm. There is an inverse relation between non-chromaticity and coherent length.

$$\text{non-chromaticity} \propto \frac{1}{L_c}$$

So, laser light has very less non-chromaticity.

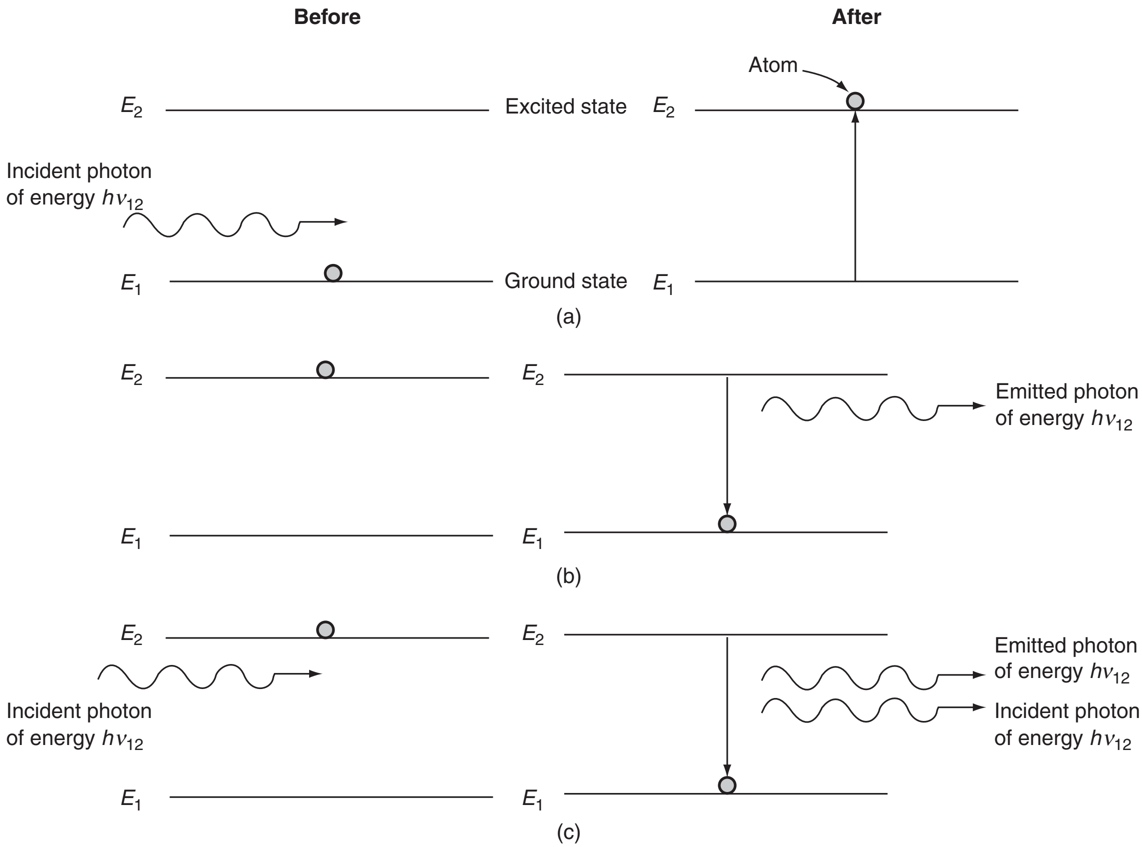
Temporal coherence: It refers to the correlation between light fields at different times at a point on the wave. If there is no change in phase over a time 't' at a point on the wave, then it is said to be coherent temporally during that time. If the phase changes many times at a point, then it is said to be incoherent. For He-Ne laser, the coherence time is 2×10^{-3} seconds whereas for sodium lamp, it is $\approx 10^{-10}$ seconds. So, there is no temporal coherence for light from sodium lamp.

(iv) Brightness: The laser beam is highly bright (intense) as compared to the conventional light sources because more light energy is concentrated in a small region. The light from an ordinary lamp comes out more or less in all directions. It has been observed that the intensity of 1mV laser light is 10,000 times brighter than the light from the sun at the earth's surface. The number of photons coming out from a laser per second per unit area is about 10^{22} to 10^{34} whereas the number of photons comes out per second per unit area of black body at 1000 K having wavelength $(\lambda) = 6000$ Å is $\approx 10^{16}$. Thus, a very hot body cannot generate the number of photons per second per unit area coming out from a laser in the visible region. Laser light is coherent, so at a time many photons are in phase and they superimpose to produce a wave of larger amplitude. The intensity is proportional to the square of amplitude. Hence, the intensity of the resultant laser beam is very high.

6.3 Spontaneous and stimulated emission

In lasers, the interaction between matter and light is of three different types. They are: absorption, spontaneous emission and stimulated emission. In these processes, two energy levels of atoms are involved. As shown in Fig. 6.1, Let E_1 and E_2 be ground and excited states of an atom. The dot in Fig. 6.1, represents an atom. Transition between these states involves absorption or emission of a photon of energy $E_2 - E_1 = h\nu_{12}$, where 'h' is Planck's constant. Now, we study these processes in detail.

Figure 6.1 (a) Absorption; (b) Spontaneous emission; (c) Stimulated emission



(a) Absorption: As shown in Fig. 6.1(a), if a photon of energy $h\nu_{12}$ ($= E_2 - E_1$) collides with an atom present in the ground state of energy E_1 then the atom completely absorbs the incident photon and makes transition to excited state E_2.

(b) Spontaneous emission: As shown in Fig. 6.1(b), an atom initially present in the excited state makes transition voluntarily on its own, without any aid of external stimulus or an agency to the ground state and emits a photon of energy $h\nu_{12}$ ($=E_2 - E_1$). This is called spontaneous emission. Different atoms of the medium emit photons at different times and in different directions. Hence, there is no phase relationship among the emitted photons, so they are incoherent. Examples for spontaneous light are glowing tube light, electric bulb, candle flame, etc.

(c) Stimulated emission: As shown in Fig. 6.1(c), a photon having energy $h\nu_{12}$ ($= E_2 - E_1$) impinges (or passes in the vicinity) on an atom present in the excited state and the atom is stimulated to make transition to the ground state and gives off a photon of energy $h\nu_{12}$. The emitted photon is in phase with the incident photon. The two photons travel in the same direction and they possess same energy and frequency. They are coherent. This type of emission is known as stimulated emission.

Differences between spontaneous emission and stimulated emission of radiation:

Spontaneous Emission	Stimulated Emission
1. Polychromatic radiation	1. Monochromatic radiation
2. Less intensity	2. High intensity
3. Less directionality, so more angular spread during propagation	3. High directionality, so less angular spread during propagation
4. Spatially and temporally incoherent radiation	4. Spatially and temporally coherent radiation
5. Spontaneous emission takes place when excited atoms make transition to lower energy level voluntarily without any external stimulation.	5. Stimulated emission takes place when a photon of energy equal to $h\nu$ ($= E_2 - E_1$) stimulates an excited atom to make transition to lower energy level.

6.4 Einstein's coefficients

To illustrate a laser, the idea of stimulated emission is essential. This can be understood in the following way: atoms absorb photons and emits photons of different frequencies. The emission may be spontaneous or stimulated. To obtain an expression that represents the ratio of spontaneous emission to stimulated emission, we consider a container having atoms and radiation. Some of the atoms present in the ground state absorb photons of energy $h\nu_{12}$ and raised to excited state (E_2) and they make spontaneous or stimulated emissions.

In steady state, let n_1 and n_2 be the number of atoms in ground state (E_1) and in excited state (E_2) per unit volume of the material. The ratio of n_1 and n_2 can be represented using Boltzmann distribution law, as:

$$\frac{n_1}{n_2} = \exp\left[\frac{E_2 - E_1}{K_B T}\right] = \exp\left[\frac{hf}{K_B T}\right] \qquad \text{\_\_\_\_\_\_\_\_\_\_ (6.1)}$$

where K_B is Boltzmann constant, f is the frequency of radiation and T is the absolute temperature of the atoms. Inside the container, radiation is present so, the number of photons per unit volume having frequencies around 'f' in unit range [i.e., radiation density] is represented as $\sigma(f)$ and is given by Planck's black body radiation law as:

$$\sigma(f) = \frac{8\pi hf^3}{c^3\left[\exp\left(\frac{hf}{K_B T}\right) - 1\right]} \qquad \text{\_\_\_\_\_\_\_\_\_\_ (6.2)}$$

where 'h' is Planck's constant and c is the velocity of light. An atom in the lower energy state E_1 gets excited to E_2 state by absorbing radiation of frequency,

$$f = \frac{E_2 - E_1}{h} \qquad \text{\_\_\_\_\_\_\_\_\_\_ (6.3)}$$

The number of such absorptions in unit volume of the material per unit time is proportional to n_1 and radiation density $\sigma(f)$. Hence, we have:

The absorption rate $= B_{12}\ n_1\ \sigma(f)$ __________ (6.4)

where B_{12} is the absorption proportionality constant. The atoms in the excited state are unstable, they make transition from excited state to ground state by making spontaneous and stimulated emissions. The number of spontaneous emissions in unit volume of the material per unit time is proportional to n_2. Hence, we have:

The spontaneous emission rate $= A_{21}n_2$ __________ (6.5)

where A_{21} is the spontaneous emission proportionality constant. Similarly, the number of stimulated emissions in unit volume of the material per unit time is proportional to n_2 and radiation density, $\sigma(f)$. Hence, we have:

The stimulated emission rate $= B_{21}n_2\sigma(f)$ __________ (6.6)

where B_{21} = stimulated emission proportionality constant.

In steady state,

$\therefore$ From Equations (6.4), (6.5) and (6.6) we write:

$$B_{12}n_1\sigma(f) = A_{21}n_2 + B_{21}n_2\sigma(f)$$

(or) $\sigma(f)\,[B_{12}n_1 - B_{21}n_2] = A_{21}\ n_2$ __________ (6.7)

$$\sigma(f) = \frac{A_{21}n_2}{B_{12}n_1 - B_{21}n_2} = \frac{A_{21}n_2}{n_2B_{21}\left[\frac{B_{12}n_1}{B_{21}n_2} - 1\right]}$$

$$= \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}}\frac{n_1}{n_2} - 1} \quad \text{__________ (6.8)}$$

Substituting Equation (6.1) in (6.8) for n_1/n_2, we have:

$$\sigma(f) = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}}\exp\frac{hf}{K_BT} - 1} \quad \text{__________ (6.9)}$$

In thermal equilibrium state, Equations (6.2) and (6.9) are equal.

so,

$$\frac{8\pi\, hf^3}{c^3\left[\exp\left(\frac{hf}{K_BT}\right) - 1\right]} = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}}\exp\left(\frac{hf}{K_BT}\right) - 1} \quad \text{__________ (6.10)}$$

Under stimulated emission, the probability of upward transitions and probability of downward transitions are equal, so:

$B_{12} = B_{21} = B$ and $A_{21} = A$ (say).

Then, Equation (6.10) becomes:

$$\frac{A_{21}}{B_{21}} = \frac{A}{B} = \frac{8\pi hf^3}{c^3} \qquad \text{\_\_\_\_\_\_\_\_} (6.11)$$

The proportionality constants A_{21}, B_{12} and B_{21} are called Einstein's A and B coefficients. From Equations (6.5) and (6.6), the ratio of spontaneous emission rate to stimulated emission rate is:

$$\frac{A_{21}\, n_2}{B_{21}\, n_2\, \sigma(f)} = \frac{A_{21}}{B_{21}\sigma(f)} = \frac{A/B}{\sigma(f)} \qquad \text{\_\_\_\_\_\_\_\_} (6.12)$$

Substituting Equations (6.2) and (6.11) in (6.12) gives:

$$= \frac{8\pi hf^3}{c^3} \bigg/ \frac{8\pi hf^3}{c^3\left[\exp\left(\frac{hf}{K_B T}\right) - 1\right]} = \exp\left(\frac{hf}{K_B T}\right) - 1 \qquad \text{\_\_\_\_\_\_\_\_} (6.13)$$

This ratio works out to be 10^{10}, thus at optical frequencies, the emission is predominantly spontaneous. So, the conventional light sources emit incoherent radiation.

6.5 Population inversion

Usually in a system the number of atoms (N_1) present in the ground state (E_1) is larger than the number of atoms (N_2) present in the higher energy state. The process of making $N_2 > N_1$ is called population inversion. Population inversion can be explained with three energy levels E_1, E_2 and E_3 of a system. Let E_1, E_2 and E_3 be ground state, metastable state and excited states of energies of the system respectively such that $E_1 < E_2 < E_3$. In a system, the population of atoms (N) in an energy level E, at absolute temperature T has been expressed in terms of the population (N_1) in the ground state using Boltzmann's distribution law

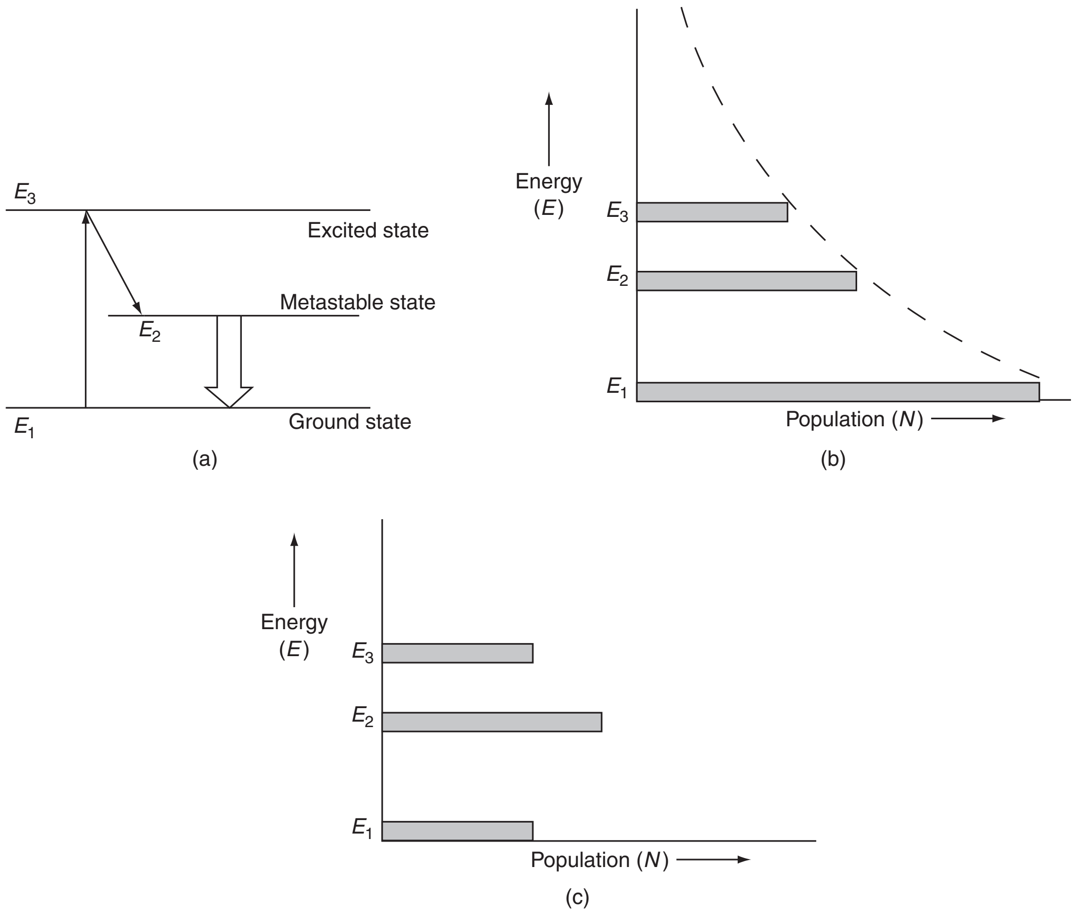
$$N = N_1 \exp(-E/K_B T) \qquad \text{where } K_B = \text{Boltzmann's constant}$$

Graphically this has been shown in Fig. 6.2(b). As shown in Fig. 6.2(a), let the atoms in the system be excited from E_1 state to E_3 state by supplying energy equal to $E_3 - E_1$ ($= h\nu$) from an external source. The atoms in E_3 state are unstable, they make downward transition in a time approximately 10^{-8} seconds to E_2 state. In E_2 state, the atoms stay over a very long duration of the order of milliseconds. So, the population of E_2 state increases steadily. As atoms in E_1 state are continuously excited to E_3 so, the population in E_1 energy level goes on decreasing. A stage will reach at which the population in E_2 state exceeds as that present in E_1 state (i.e., $N_2 > N_1$). This situation is known as population inversion. Graphically the population inversion has been shown in Fig. 6.2(c).

Conditions for population inversion are:

(a) The system should possess at least a pair of energy levels ($E_2 > E_1$), separated by an energy equal to the energy of a photon ($h\nu$).
(b) There should be a continuous supply of energy to the system such that the atoms must be raised continuously to the excited state.

Figure 6.2 (a) Population inversion between E_1 and E_2 energy levels; (b) population under thermal equilibrium and (c) population inversion of E_2 with respect to E_1



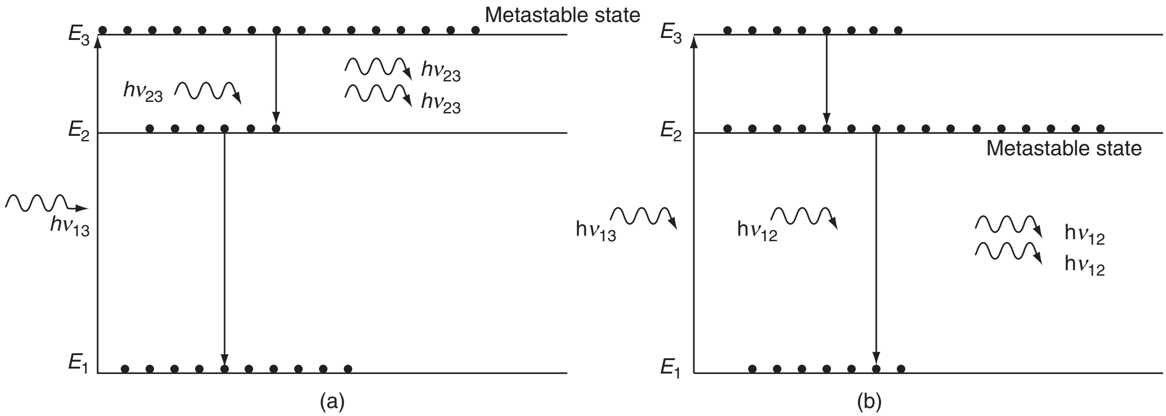
Population inversion can be achieved by a number of ways. Some of them are: (i) optical pumping (ii) electrical discharge (iii) inelastic collision of atoms (iv) chemical reaction and (v) direct conversion.

In a laser, if the active medium is a transparent dielectric, then optical pumping method is used. If the active medium is conductive, then electric field is used to produce population inversion. Few of the above pumping methods are explained below.

Optical Pumping: To explain optical pumping, we consider the three energy levels of atoms as shown in Fig. 6.3(a) and 6.3(b).

The transitions of atoms in these energy levels and laser emission has been explained in two ways. (i) As shown in Fig. 6.3(a), an atom present in the ground state (E_1 energy level) absorbs a photon of energy equal to $h\nu_{13}$ and occupies E_3 energy level.

Figure 6.3 Three-level laser energy levels



If the atoms makes transition from E_3 to E_2 energy level slowly and E_2 to E_1 fastly, then the number of atoms in E_3 energy level will be more than in E_2 energy level. i.e., population inversion ($N_3 > N_2$) exists between E_3 and E_2 energy levels. The energy level E_3 is called metastable state. An external photon of energy $h\nu_{23}$ ($=E_3 - E_2$) stimulates an atom in E_3 level and hence stimulated emission takes place. So, the photon of energy $h\nu_{23}$ acts as a laser light. The atoms present in E_2 energy level makes non-radiative transition to E_1 energy level.

(ii) As shown in Fig. 6.3(b), an atom present in the ground state (E_1 energy level) absorbs a photon of energy $h\nu_{13}$ and excited to E_3 energy level. The atoms will remain very short duration ($<10^{-8}$ sec) in E_3 energy level and makes transition to E_2 energy. This transition is non-radiative. In E_2 energy level, the atoms will remain for longer duration than in E_3 energy level. By continuous supply of energy $h\nu_{13}$, the number of atoms in E_2 energy level go on increasing and the number of atoms in E_1 energy level is reduced. Hence, population inversion exists between E_2 and E_1 energy levels i.e., $N_2 > N_1$. Now, an external photon of energy $h\nu_{12}$ can make stimulated emission. Hence, a laser beam of photon energy $h\nu$ is obtained.

In electrical pumping, the applied electric field caused ionization of the medium and raises to the excited state. This has been used in gas lasers.

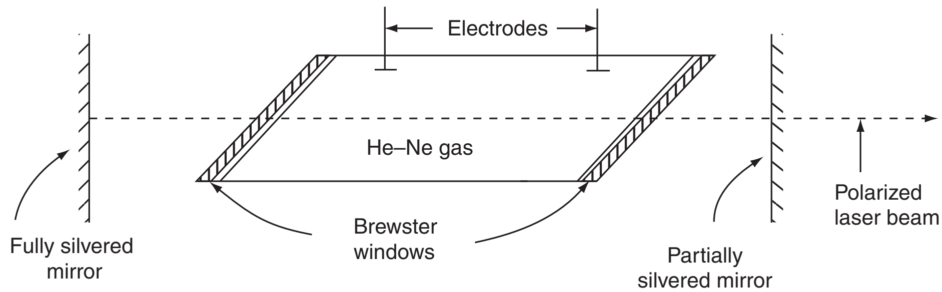
Direct conversion of electric energy into light energy has been used in semiconductor lasers.

6.6 Helium–Neon gas [He–Ne] laser

Helium–Neon gas laser is a continuous four level gas laser. It consists of a long, narrow cylindrical tube made up of fused quartz. The diameter of the tube will vary from 2 to 8 mm and length will vary from 10 to 100 cm. Flat quartz plates are sealed at the ends of the tube, the plates are sealed at Brewster angle with the axis of the tube to obtain polarized laser light as shown in Fig. 6.4. So, the plates are called Brewster windows. The tube is filled with helium and neon gases in the ratio of 10:1. The partial pressure of helium gas is 1 mm of Hg and neon gas is 0.1 mm of Hg, so that the pressure of the mixture of gases inside the tube is nearly 1 mm of Hg.

Laser action is due to the neon atoms. Helium is used for selective pumping of neon atoms to upper energy levels. Two electrodes are fixed near the ends of the tube to pass electric discharge through the gas. Two optically plane mirrors are fixed at the two ends of the tube normal to its axis. One of the mirrors is

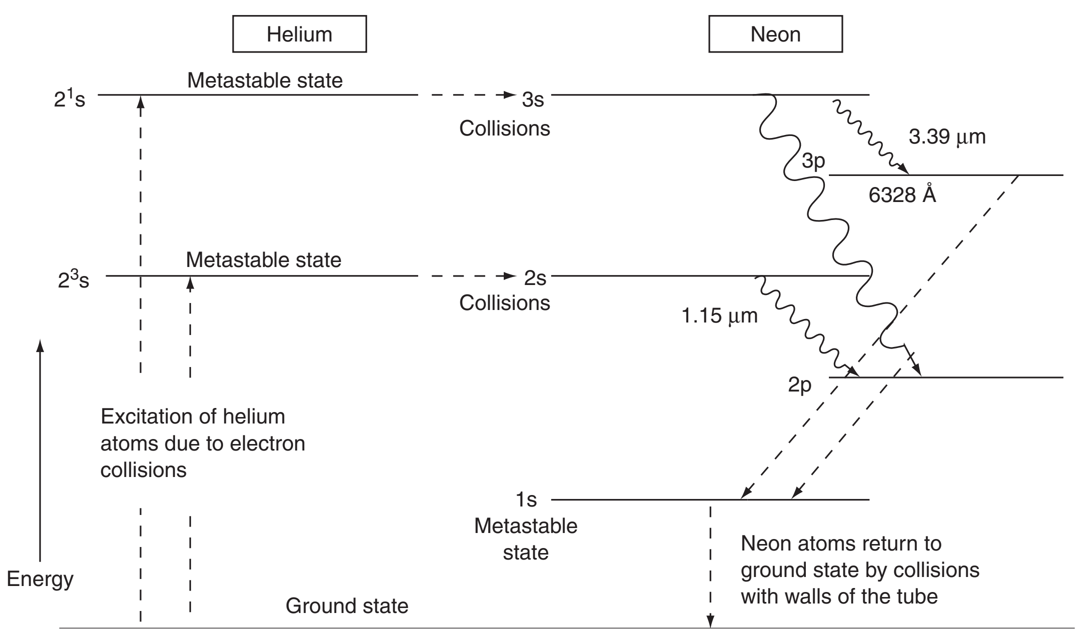
Figure 6.4 Helium–Neon gas laser



fully silvered so that nearly 100% reflection takes place and the other is partially silvered so that 1% of the light incident on it will be transmitted. Optical resonance column is formed between these mirrors.

Working: When a voltage of about 1000 V is applied between the electrodes, then electric discharge takes place through the gas in the tube. The free electrons accelerate towards the positive electrode. In their journey, some of these electrons collide with the majority helium gaseous atoms in the tube. When a fast-moving electron collides with a ground state He atoms then the helium atoms are pumped to two metastable energy levels 2^1s and 2^3s of helium as shown in Fig. 6.5. In the metastable state, the atoms remain relatively long time. So, more number of helium atoms will be present in metastable state than in ground state, which leads to an increase of population in each of these metastable states.

Figure 6.5 He–Ne energy level diagram



Inside the tube, the helium atoms present in metastable states may make collisions with the neon atoms present in the ground state and excite them to 2s and 3s levels. During collisions, the metastable helium atoms transfer their energy to ground state neon atoms and the helium atoms come back to the ground state. During collision, resonance transfer of energy form He to Ne atoms takes place because the 2^1s and 2^3s energy levels of helium atoms are very close with 3s and 2s energy levels of neon. Thus, the population inversion in neon atoms takes place. The excited neon atoms transit to ground state in three different ways leading to three lasers of different wavelengths. They are: (a) transition from 3s level to 3p level gives rise to radiation of wavelength 3.39 μm, this lies in the infrared region (b) transition from 3s level to 2p level gives rise to visible radiation of wavelength 6328 Å, this lies in red region and (c) the transition from 2s level to 2p level gives rise to a wavelength of 1.15 μm, this again lies in the infrared region. The atoms in 3p and 2p level undergo spontaneous transitions to 1s level, this is a metastable state [3s and 2s levels are not metastable states]. The photons emitted by the atoms coming down from 3p or 2p level to 1s level are likely to excite the 1s atoms back to 3p or 2p levels. This affects the population inversion in 3s and 2s levels. The atoms in 1s level return back to the ground level mainly by collisions with the walls of the discharge tube. This effect makes the gain of He-Ne laser to be inversely proportional to the diameter of the discharge tube, so the discharge tubes are made only to a few millimetres of diameter. The mirrors placed out side the tube produces optical pumping in the resonance column inside the tube, which enhances stimulated emissions. Red laser light comes out of the partially silvered mirror.

6.7 Ruby laser

Ruby laser is a solid state pulsed, three-level laser. It consists of a cylindrical-shaped ruby crystal rod of length varying from 2 to 20 cms and diameter varying from 0.1 to 2 cms. The end faces of the rod are highly flat and parallel. One of the faces is highly silvered and the other face is partially silvered so that it transmits 10 to 25% of incident light and reflects the rest so as to make the rod-resonant cavity. Basically, ruby crystal is aluminium oxide [Al_2O_3] doped with 0.05 to 0.5% of chromium atoms. These chromium atoms serve as activators. Due to the presence of 0.05% of chromium, the ruby crystal appears in pink colour. The ruby crystal is placed along the axis of a helical Xenon or Krypton flash lamp of high intensity. This is surrounded by a reflector as shown in Fig. 6.6. The ends of the flash lamp are connected to a pulsed high-voltage source, so that the lamp gives flashes of an intense light.

Each flash of light lasts for several milliseconds. The ruby rod absorbs the flashes of light to excite chromium ions [Cr^{3+}] to higher energy levels. During the course of flash, enormous amount of heat is produced. The ruby rod is protected from the heat by enclosing it in a hollow tube through which cold water is circulated [not shown in the Figure]. The chromium ions are responsible for the stimulated emission of radiations whereas aluminium and oxygen ions are passive.

The emission of radiations by chromium atoms can be explained with the help of energy level diagram as shown in Fig. 6.7. The energy level diagram of a solid consists of energy bands. As large number of the chromium ions absorb the radiations of wavelength around 5500 Å and 4000 Å emitted by the flash lamp and get excited to $^4F_1$ and $^4F_2$ energy levels from ground state. The chromium ions remain for about 10^{-8} to 10^{-9} seconds in these energy levels and makes non-radiative transition to the metastable state ^2E, consisting of a pair of energy levels. In metastable state, the chromium ions remain for longer duration of the order of milliseconds. So, population inversion takes place between metastable and ground state. As a result, stimulated emission takes place and the chromium ions translate from metastable to ground state. The transitions give rise to the emission of light of wave lengths 6929 Å and 6943 Å, respectively. In these 6929 Å, wavelength radiation is

Figure 6.6 Ruby laser

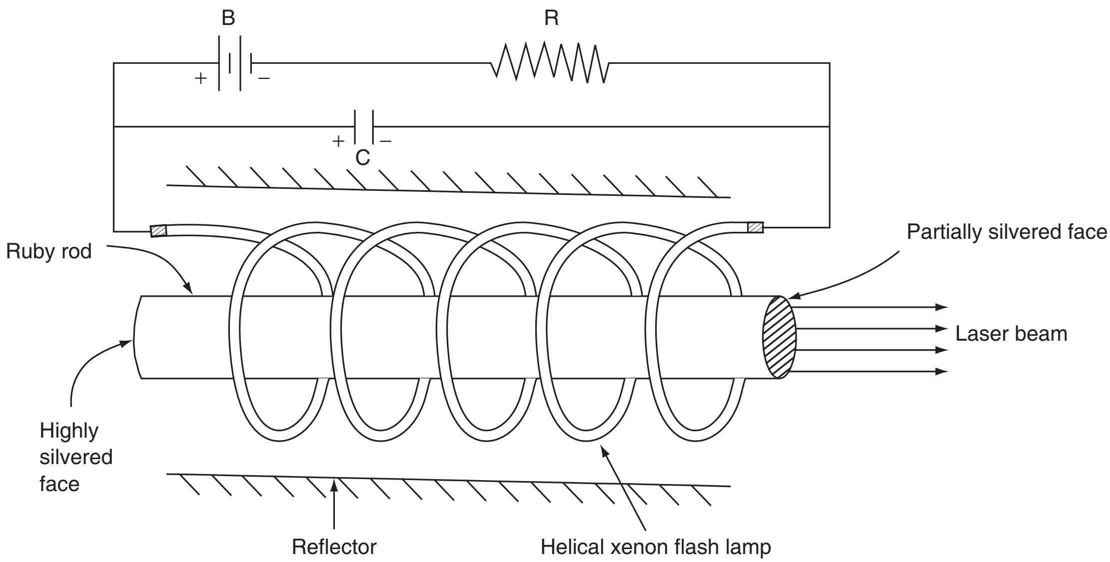
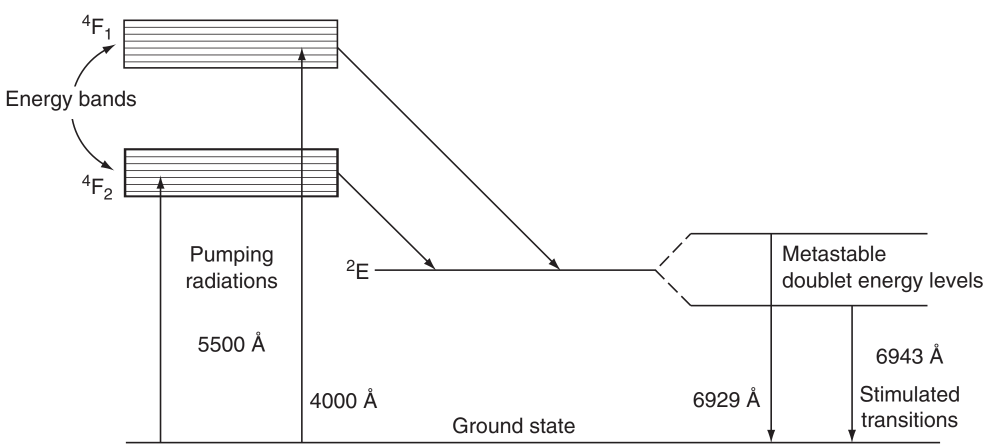


Figure 6.7 Energy level diagram of chromium ions in a ruby crystal



very weak in intensity and the laser radiation is mostly due to 6943 Å wavelength radiation. The spontaneously emitted initial photons would travel in all directions, of these, those travelling parallel to the axis of the rod would be reflected at the ends and pass many times through the amplifying medium and stimulates the atoms in metastable state. The output of this laser consists of a series of laser pulses for a duration of microseconds or less.

6.8 Semiconductor lasers

A highly doped p-n junction diode made up of direct band gap semiconductor material under forward bias emits photons from the junction due to the recombination of conduction band electrons and valence band holes. Example for direct band gap semiconductor is GaAs. During recombination a conduction band electron crosses the energy gap (E_g) and combines with a hole present in the valence band. A photon of energy, $h\nu$ equal to E_g is released. Hence $E_g = h\nu = hc/\lambda$. Where h = Planck's constant = 6.63×10^{-34} J-S; C = velocity of light = 3×10^8 m/s and λ = wavelength of emitted photon. In semiconductors, p-n junction is the active region to produce laser radiation. To produce laser radiation two conditions must be satisfied: (1) population inversion and (2) optical feedback. Population inversion means there must be a region of the device in which large density of free electrons in the bottom energy levels of conduction band and large density of holes in the top energy levels of valence band exists. This is obtained with high doping concentration. Optical feedback is obtained by clearing or by polishing the ends of p-n junction at right angles to the junction layer. Forward-biased current is slowly increased through the junction. At low current densities, spontaneous emission takes place. Above threshold current density-stimulated emission takes place.

Homo- and Heterostructure lasers: If the energy gap width of the semiconductor material on one side of p-n junction is the same as that on the other side of the junction, then such a semiconductor laser is known as homostructure laser. On the other hand, if the energy gap width of the semiconductor material on one side of p-n junction is different from that on the other side of the junction, then such a semiconductor laser is known as heterostructure laser. The basic structure of a p-n junction laser is shown in Fig. 6.8.

A pair of opposite parallel faces of the p-n junction laser are polished to provide optical feedback and the other two opposite faces are roughened to eliminate lasing in that direction.

The band diagram of a heavily doped homostructure p-n junction laser is shown is Fig. 6.9(a). In the n^+ region, the Fermi level lies within the conduction band and in the P^+ region, the Fermi level lies in the valence band. The junction is forward-biased such that the biasing voltage is equal to the energy gap voltage (E_g/e), then the electrons and holes are injected across the junction and population inversion takes place in the active region.

The band diagram after forward biasing is shown in Fig. 6. 9(b).

Figure 6.8 Semiconductor p-n junction laser

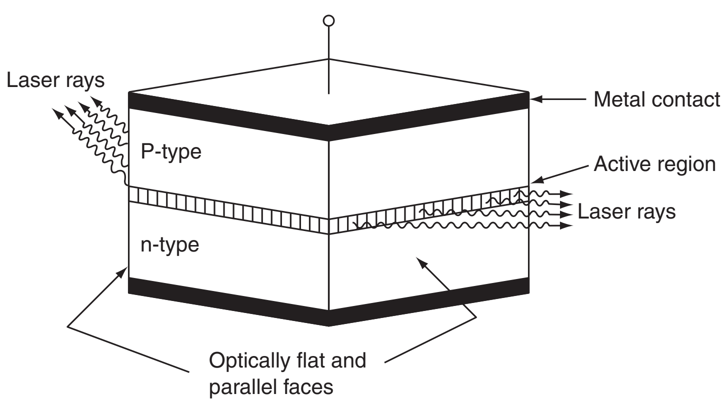
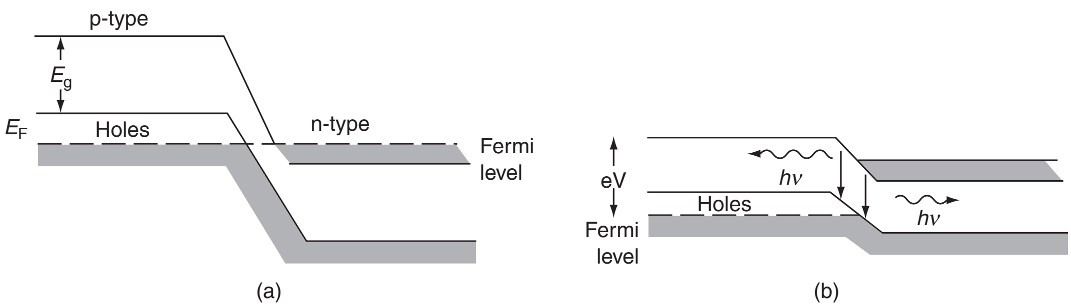


Figure 6.9 (a) Band diagram of a heavily doped p-n junction in equilibrium and (b) band diagram of a heavily doped p-n junction with forward bias



When the biasing current is low, then spontaneous emission takes place in all directions. As the biasing current reaches threshold value, then stimulated emission occurs and a monochromatic and highly directional beam of light is emitted from the junction.

In case of homojunction lasers [for example GaAs p-n junction], the threshold current density (J_{th}) increases rapidly with raise of temperature. At room temperature [300K], the threshold current density is about 5.0×10^4 A/cm^2. This large current density leads to serious difficulties in operating the laser continuously at room temperature. Threshold current densities are of the order of 10^3 A/cm^2 in heterojunction lasers. These lasers are built using epitaxial techniques as shown in Fig. 6.6.

In the structure shown in Fig. 6.10(a), there is only one heterojunction and that shown in Fig. 6.10(b), there are two heterojunctions. In $Al_xGa_{1-x}As$, x represents mole fraction.

The variation of threshold current density with temperature is very much less in double heterostructure laser when compared to homostructure and single heterostructure laser. The above semiconductor lasers are broad area lasers. Because, in the complete area of p-n junction, lasing action takes place. To reduce the operating currents to a large extent, heterostructure strip geometry lasers are used. Figure 6.11 shows two such strip geometries.

The various layers shown in the above structures are fabricated by epitaxial growth. The oxide layer in the structure shown in Fig. 6.11(a) is an insulating layer. A thin strip of oxide layer at the centre has been removed by chemical etching techniques and a metal layer was deposited. On biasing, current passes under the strip contact only, because the oxide layer insulates the remaining region. Lasing takes place under the strip in the active layer. Instead of an oxide layer, proton bombardment is carried at the top surface of the structure as shown in Fig. 6.11(b). Proton bombardment produces high resistance; this bombardment has been carried except along a stripe at the centre of active layer. The strip widths

Figure 6.10 (a) Single heterostructure laser and (b) double heterostructure laser

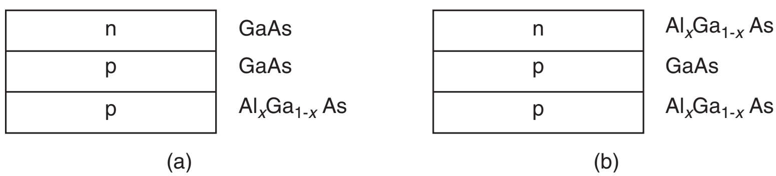
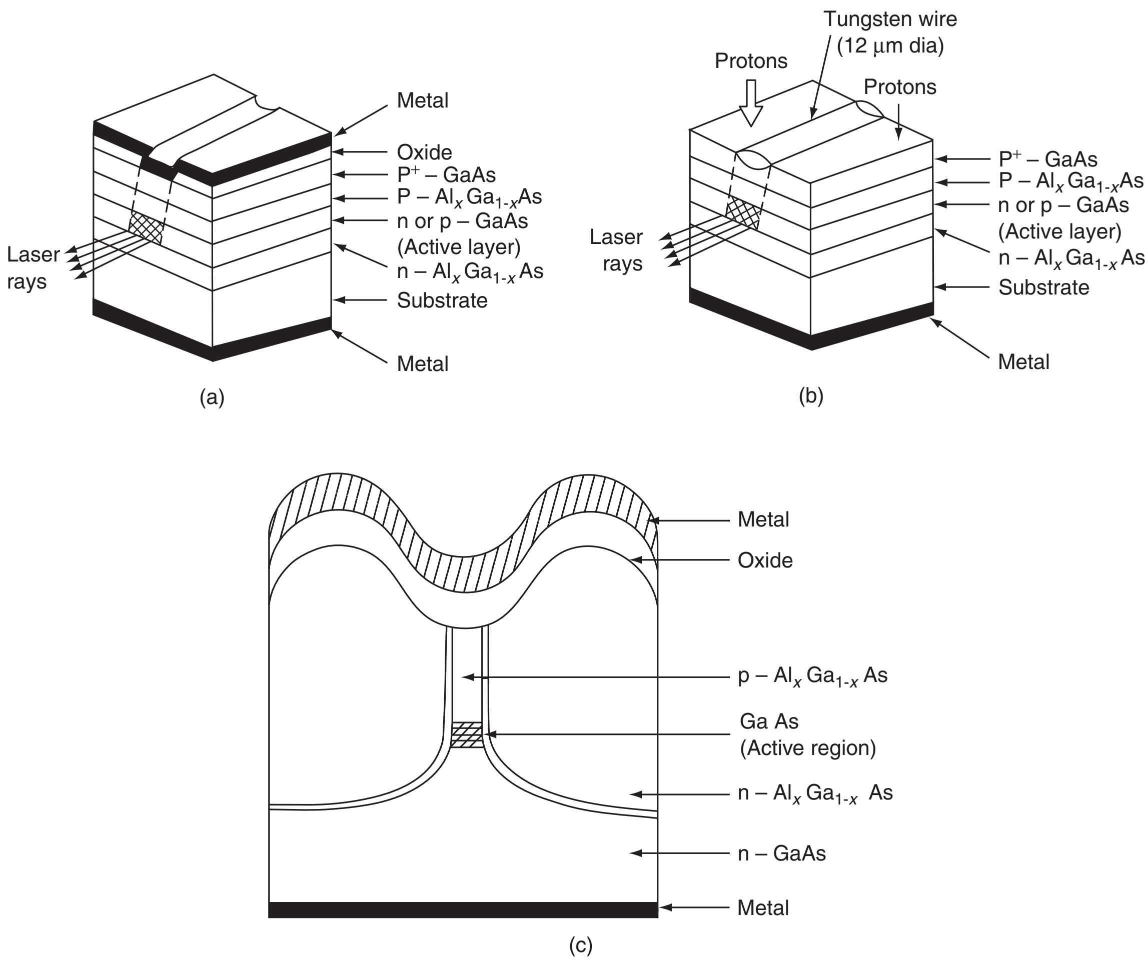


Figure 6.11 (a) Oxide-coated heterostructure strip geometry laser; (b) proton-bombarding heterostructure strip geometry laser and (c) buried heterostructure laser



vary from 5 to 30 μm. The advantages of stripe geometry are: (1) reduction of operating current and (2) improve response time due to small junction capacitance.

Lasing action can be obtained with extremely small currents by using buried heterostructure laser shown in Fig. 6.11(c). The active region in this structure is completely surrounded by higher band gap, lower refractive index material so, all those photons which are produced in the active region and whose energies are less than energy gap of the surrounding material are transmitted without absorption. Due to lower refractive index of the surrounding material, the rays bend less when come out of the device. The cross-sectional area of the active region is as small as 1 μm^2. So, the threshold current is as low as 15 mA.

6.9 Carbon dioxide laser

(i) Introduction: The carbon dioxide laser was invented by C. Kumar. N. Patel in the year 1963. This laser uses a mixture of carbon dioxide [CO_2], Nitrogen [N_2] and Helium [He] gases in the active medium of laser. The lasing action is from carbon dioxide molecules. They are the active molecules in the laser. The CO_2 molecule is a linear symmetric molecule with carbon atom at the centre and oxygen atoms are on each side of the carbon atom. The lasing is due to the transitions of CO_2 molecules in between vibrational–rotational levels. The CO_2 molecules possess three different types of vibrational modes and each vibrational mode possesses a number of rotational modes. The vibrational modes are described as shown in Fig. 6.12.

In symmetric stretching mode, the carbon atom is stationary and the oxygen atoms symmetrically move away or approaches the carbon atom.

In the bending mode, some atoms (not all) move perpendicular to the molecular axis. In asymmetric stretching mode, both the oxygen molecules move in one direction along the molecular axis and carbon move in opposite direction. [The state of a vibrating molecule is represented by a set of three vibrational quantum numbers, labeled as (nm^lp), where n is the frequency of the photon emitted due to symmetric stretch, m is the frequency of the photon emitted due to bending and p is the frequency of the photon emitted due to asymmetric stretch. The bending vibration is doubly degenerate, i.e., it can occur both in the plane of the figure and the plane perpendicular to it. The superscript l represents the angular momentum of this vibration with respect to molecular axis. For example, (02°0) shows the two vibrations combine to give an angular momentum $l = 0$].

(ii) Construction: As shown in Fig. 6.13, one of the CO_2 laser consists of a long tube of about 5 m long and 2.5 cm diameter. The output power of this laser is approximately, proportional to the length of the tube. The ends of the tube is closed with alkali halide [NaCl] Brewster windows. Outside the ends of the tube, confocal

Figure 6.12 Fundamental modes of vibration of CO_2 molecule

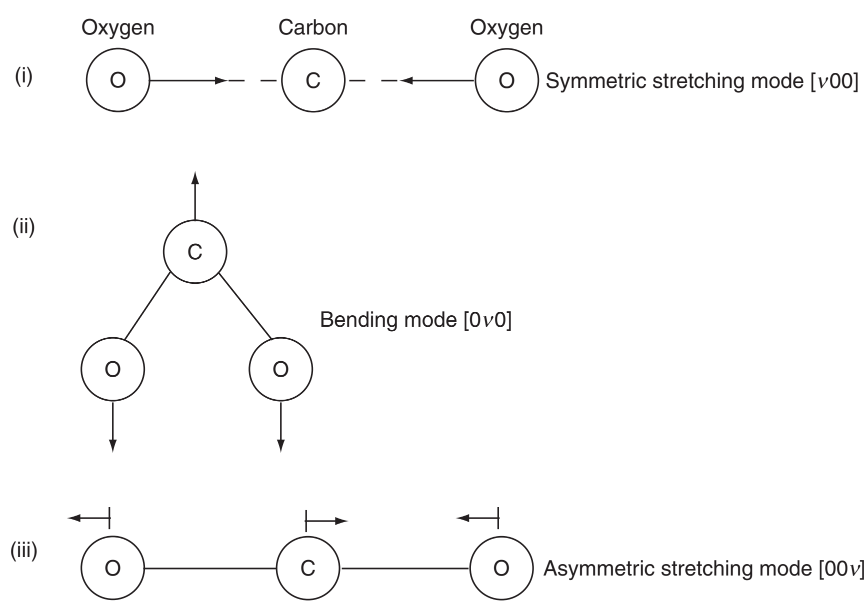
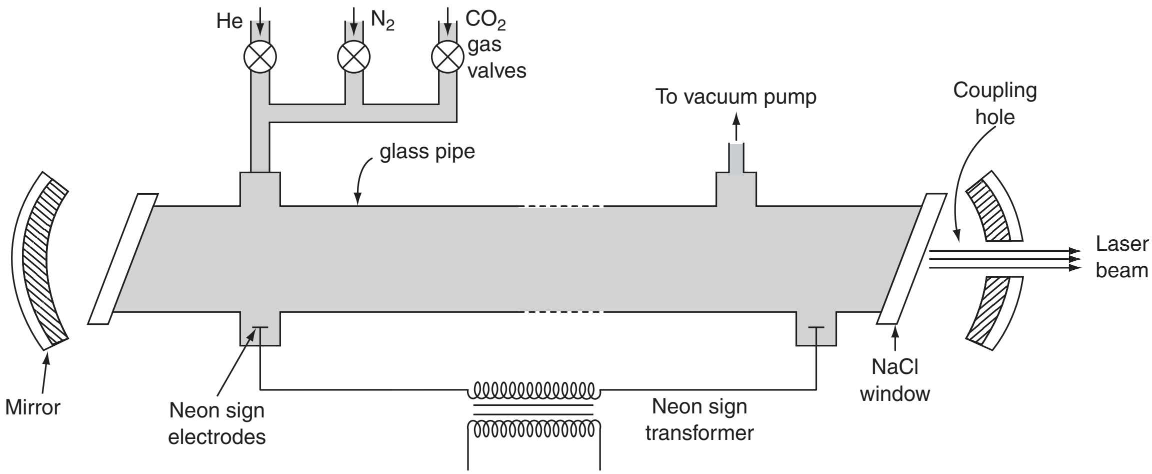


Figure 6.13 CO_2 laser



silicon mirrors coated with aluminium are arranged. This forms the resonant cavity. The gases CO_2, N_2 and He are allowed into the tube through gas valves. Inside the tube, these gases combine and continuously pass through it. During discharge, the gases may dissociate and contaminate the laser, so continuous flow of gases is maintained in the tube.

The pressures of the gases in the tube are $P_{He} \approx 7$ torr; $P_{N_2} \approx 1.2$ torr and $P_{CO_2} \approx 0.33$ torr.

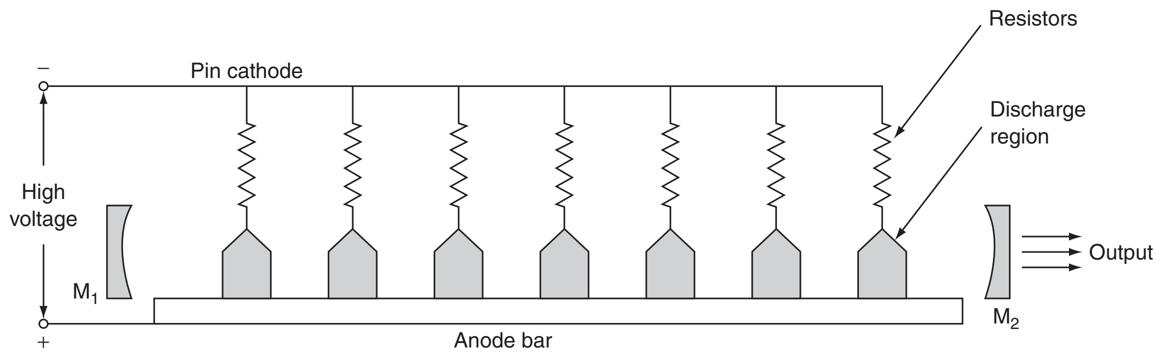
The optimum value of pressure tube diameter product is around 33 torr mm. The purpose of N_2 gas in the tube is to produce high population inversion in CO_2 molecules. Here, resonance transfer of energy from N_2 gas molecules to CO_2 gas molecules takes place in the tube. To avoid population in the lower laser levels by thermal excitation, it is necessary that the temperature of CO_2 should be low. For this purpose, helium gas is passed through the tube along CO_2 and N_2 gases, because helium gas possesses high thermal conductivity and helps to conduct heat away to the walls, keeping CO_2 temperature low. Thus N_2 helps to increase the population of upper level and helium helps to depopulate the lower level.

(iii) Working: Sufficiently high voltage of the order of 8 KV per metre length of the tube must be maintained to get discharge. Two different configurations are available for high output power. They are TEA and Gas dynamic laser.

(a) TEA [Transverse Excitation Atmospheric] CO_2 laser: The output power of the laser can be increased by increasing gas pressure. At atmospheric pressure, to initiate and maintain electric discharge, 12 KV per cm is required. In longitudinal configuration with 1 m or above length tubes, it is not possible to apply such high electric fields. In TEA laser, the discharge is arranged to take place at a number of points in a direction transverse to laser cavity, as shown in Fig. 6.14.

(b) Gas dynamic laser: In this laser, population inversion is obtained through thermodynamic principles rather than discharge. The gas mixture containing N_2 and CO_2 is heated, compressed and allow to expand into low-pressure region. At high temperatures, the N_2 molecules reaches to the higher vibrational levels and after expansion into low-pressure region, the N_2 molecules makes resonant collisions with CO_2 molecules. Then, the CO_2 molecules makes transition to (001) state and produce population inversion. This laser produce output powers in excess of 100 KW.

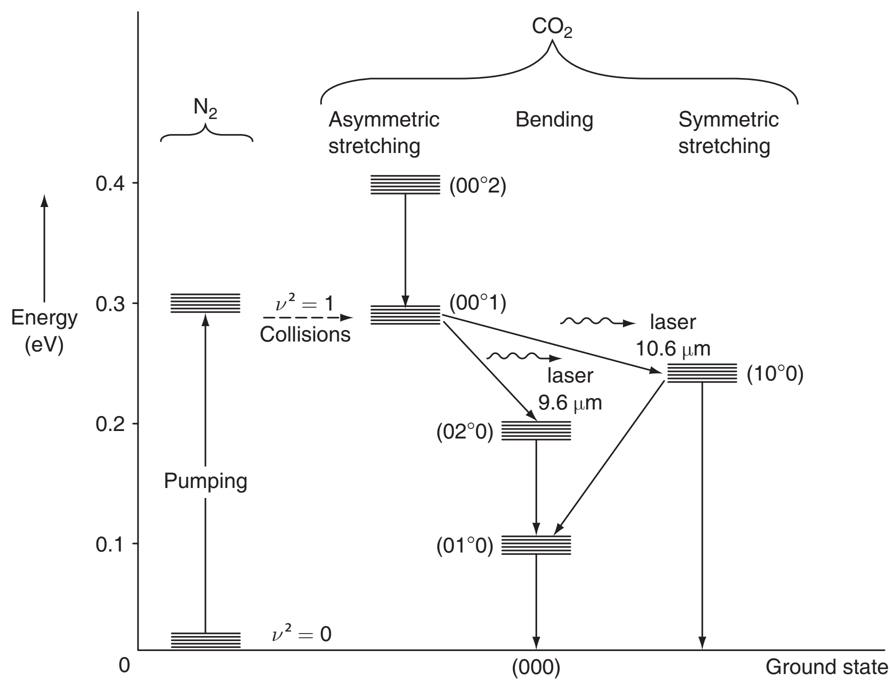
Figure 6.14 TEA laser



The emission of laser radiation from CO_2 molecules has been explained with the help of vibrational-rotational energy levels in the following way.

The electric discharge in CO_2 laser may excite CO_2 and N_2 molecules to higher vibrational rotational levels by electron impacts. The electronic collision cross-section of CO_2 for the excitation to the level

Figure 6.15 CO_2 laser energy levels



(00°1) is very large, this is the metastable state. This level is populated by: (i) non-radiative transition from the upper excited levels such as (00°2) of CO_2 and (ii) the collision of N_2 molecules present in $\nu'' = 1$ level with CO_2 molecules lead to resonance transfer of energy. Because the $\nu'' = 1$ level of N_2 molecules and (00°1) levels of CO_2 are nearly at the same energy and the lifetime of $\nu'' = 1$ is quite large [0.1 s at 1 torr]. Population inversion exists between (00°1) and (10°0), (02°0) levels. Two laser transitions takes place between these levels: (i) (00°1) to (10°0) gives laser radiation of wavelength 6.6 μm and (ii) (00°1) to (02°0) gives laser radiation of wavelength 9.6 μm. Of these 6.6 μm waves are intense, its power output is of 10 KW, efficiency being 30%, this is quite large and 9.6 μm waves are weak. The lower levels (10°0), (02°0) and (01°0) are very close. The CO_2 molecules present in (10°0), (02°0) and (01°0) levels may make non-radiative transition to the ground state quickly by colliding with He atoms (Fig. 6.15). In this process, energy dissipation takes place in the form of heat.

6.10 Applications of lasers

Lasers find applications in various fields of science and technology. They are described below:

(1) In communications:

(a) Lasers are used in optical fibre communications. In optical fibre communications, lasers are used as light source to transmit audio, video signals and data to long distances without attenuation and distortion.

(b) The narrow angular spread of laser beam recommends that laser beam can be used for the communication between earth and moon or to other satellites.

(c) As laser radiation is not absorbed by water, so laser beam can be used in under water [inside sea] communication networks.

(2) Industrial applications: Lasers are used in metal cutting, welding, surface treatment and hole drilling.

(a) When a laser beam is focussed on a very small area, then laser light energy is converted into heat energy, so, the material may be heated, melted and evaporated. Using these techniques, holes can be drilled in steel, ceramics, diamond and alloys. Using lasers, controlled orifices and aerosol nozzles are drilled with controlled precision. Holes of micron order can be easily drilled using lasers. These techniques are used in cutting materials such as metal sheets and cloths. In mass production of stitched clothes, the cutting in the desired dimension is done by stock filing a large number of cloth material and cutting them all at once by exposing a laser beam.

Using lasers, cutting can be obtained to any desired shape and the cut surface is very smooth.

(b) Welding has been carried by using laser beam. A laser beam is exposed to the place where welding has to be done, at that place the material melts due to the heat produced by the beam and on cooling the material makes a stronger joint.

(c) Dissimilar metals can be welded and microwelding is done with great ease.

(d) Laser beam is used in selective heat treatment for tempering the desired parts in automobile industry.

(e) Lasers are widely used in electronic industry in trimming the components of ICs.

(3) Medical applications:

(a) Lasers are used in eye surgery, especially in detached retina. Under certain abnormal conditions, the retina may get detached from the choroid, this results blindness at the detached part of the retina. The

retina can be attached to the choroid by heating it over a number of spots almost of the size of tissues. The heating can be achieved by focusing a laser beam of predetermined intensity on to the retina. The laser beam reaches the right spot where the welding of the retina to the choroid occurs. The flash of laser beam lost in short time (1 milli second). The patient does not feel any pain, so anaesthesia is not required.

(b) Lasers are used for treatments such as plastic surgery, skin injuries and to remove moles and tumours developed in skin tissue.

(c) Lasers are used in stomatology—the study of mouth and its disease. Where a laser beam is used for selective destroying, the part of the tooth affected by caries. Mouth ulcers can be cured by exposing it to a laser beam.

(d) Laser radiation is sent through optical fibre to open the blocked artery region, here the laser rays burn the excess growth in the blocked region and regulates blood flow without any requirement for bypass surgery.

(e) Lasers are used to destroy kidney stones and gall stones. The laser pulses are sent through optical fibres to the stoned region. The laser pulses break the stones into small pieces.

(f) Lasers are used in cancer diagnosis and therapy.

(g) Lasers are used in blood loss less surgery. During operation, the cut blood veins are fused at their tips by exposing to infrared laser light, so that there is no blood loss.

(h) Lasers are used to control haemorrhage.

(i) Using organ and CO_2 lasers, liver and lung treatment can be carried out.

(j) Lasers are used in endoscopes to detect hidden parts.

(k) Laser Doppler velocimetry is used to measure blood velocity in the blood vessels.

(4) Military applications: The various military applications are:

(a) Death rays: By focusing high-energetic laser beam for few seconds to aircraft, missile, etc. can be destroyed. So, these rays are called death rays or war weapons.

(b) Laser gun: The vital part of enemy body can be evaporated at short range by focusing a highly convergent laser beam from a laser gun.

(c) LIDAR [Light Detecting And Ranging]: In place of RADAR, we can use LIDAR to estimate the size and shape of distant objects or war weapons. The difference between RADAR and LIDAR is that, in case of RADAR, radio waves are used where as in case of LIDAR light is used.

(5) In computers: By using lasers, a large amount of information or data can be stored in CD-ROM or their storage capacity can be increased. Lasers are also used in computer printers.

(6) In thermonuclear fusion: To initiate nuclear fusion reaction, very high temperature and pressure is required. This can be created by concentrating large amount of laser energy in a small volume. In the fusion of deuterium and tritium, irradiation with a high-energy laser beam pulse of 1 nanosecond duration develops a temperature of 10^{17} °C, this temperature is sufficient to initiate nuclear fusion reaction.

(7) In scientific research:

(a) Laser beam can initiate or fasten chemical reactions. Laser beam helps us to study the nature of chemical bonds. An intense laser beam can break molecules.

(b) Lasers are used in counting of atoms in isotope separation and to separate isotopes of uranium.

(c) Lasers are used to estimate the size and shape of biological cells such as erythrocytes.

(d) Lasers can be used in air pollution, to find the size of dust particles.

(e) Lasers are used in holography for recording and reconstruction of a hologram. Using holograms, the three-dimensional images of objects can be recorded.

(f) To measure the constantly changing distance between moon and earth by astronomers. This gives the day-to-day changes in the rotation of earth about its axis and slight wobbles.
(g) In plastic industries, polymers are obtained by irradiating monomers. During laser irradiation, the monomers are united to form polymers.
(h) By using lasers, the stimulated Raman spectrum is obtained for small biological samples.
(i) Lasers are used to develop hidden finger prints and to clean delicate pieces of art.

Formula

1. $$\frac{\text{Spontaneous emission}}{\text{Stimulated emission}} = \frac{A/B}{\sigma(f)} = \exp\left(\frac{hf}{K_B T}\right) - 1$$

Solved Problems

1. *A semiconductor diode laser has a peak emission wavelength of 1.55* μm. *Find its band gap in* eV.

(Set-2–May 2008)

Sol: Energy gap of semiconductor, E_g = energy of emitted photon, $h\nu$

$E_g = h\nu = hc/\lambda$ where c = velocity of light = 3×10^8 m/s

Wavelength, $\lambda = 1.55\ \mu\text{m} = 1.55 \times 16^{-6}$ m

Energy gap, $E_g = ?$

$$E_g = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.55 \times 10^{-6}}\ \text{J}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.55 \times 10^{-6} \times 1.6 \times 10^{-19}}\ \text{eV} = 0.8\ \text{eV}$$

2. *Calculate the wavelength of emitted radiation from GaAs which has a band gap of 1.44* eV.

(Set-3–May 2008)

Sol: Energy gap of semiconductor, Eg = $h\nu$

h = Planck's constant = 6.63×10^{-34} J-S

$$E_g = h\nu = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{E_g}$$

$E_g = 1.44\ \text{eV} = 1.44 \times 1.6 \times 10^{-19}$ J

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.44 \times 1.6 \times 10^{-19}} = 8633 \times 10^{-10}\ \text{m} = 8633\ \text{Å}$$

3. *Prove that the stimulated emission will not takes place for Sodium D line at 300°C and also at optical frequencies in thermal equilibrium.*

Sol: From Einstein's coefficients, we know

$$\frac{\text{Stimulated emission}}{\text{Spontaneous emission}} = \frac{1}{e^{h\nu/K_BT} - 1} = \frac{1}{e^{hc/\lambda K_BT} - 1}$$

For Sodium D line, λ = 5890 Å = 5890 × 10^{-10} m
Temperature of sodium, T = 300°C = (300 + 273) K = 573 K
Boltzmann constant, K_B = 1.38 × 10^{-23} J/K
Planck's constant, h = 6.63 × 10^{-34} J.S
Substituting these values in the above equation, we have

$$= \frac{1}{\exp\left[\dfrac{6.63\times10^{-34}\times3\times10^{8}}{5890\times10^{-10}\times1.38\times10^{-23}\times573}\right]-1}$$

$$= \frac{1}{\exp\left[\dfrac{6.63\times3}{.589\times1.38\times.573}\right]-1} = \frac{1}{\exp 43.7057-1}$$

$$= \frac{1}{3.5\times10^{18}-1} \approx \frac{1}{3.5\times10^{18}}$$

The above ratio shows that the spontaneous emissions are very large compared to stimulated emissions. For optical frequencies, the wavelength is nearly 5000 Å and in thermal equilibrium means at room temperature (T = 300 K).

Hence,

$$\frac{\text{Stimulated emission}}{\text{Spontaneous emission}} = \frac{1}{e^{h\nu/K_BT} - 1} = \frac{1}{e^{hc/\lambda K_BT} - 1}$$

Substituting the values in the above equation,

$$= \frac{1}{\exp\left[\dfrac{6.63\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}\times1.38\times10^{-23}\times300}\right]-1}$$

$$= \frac{1}{\exp\left[\dfrac{6.63\times3}{0.5\times1.38\times0.3}\right]-1} = \frac{1}{\exp 96.087-1}$$

$$= \frac{1}{5.37\times10^{41}-1} \approx \frac{1}{5.37\times10^{41}}$$

The above ratio indicates that the spontaneous emission is very large compared to stimulated emission. Hence, we say that the stimulated emission is not possible at optical frequencies in thermal equilibrium.

Multiple-choice Questions

1. Laser action is found in ___________ semiconductor.

   (a) direct band gap
   (b) indirect band gap
   (c) germanium
   (d) silicon

2. In computer, printers ___________ laser is used.

   (a) He–Ne gas
   (b) ruby
   (c) semiconductor
   (d) CO_2

3. Laser radiation is __________ .

(a) highly monochromatic (b) partially monochromatic
(c) white light (d) none

4. Under population inversion, the number of atoms in the higher energy state is __________ than in the lower energy state.

(a) lesser (b) larger (c) both a and b (d) none

5. Laser radiation is __________ .

(a) highly directional (b) monochromatic
(c) coherent and stimulated (d) all

6. In conventional light sources, __________ .

(a) different atoms emit radiation at different times
(b) there is no phase relation between the emitted photons
(c) different atoms emit photons in different directions
(d) all

7. In laser sources, __________ .

(a) photons emitted by different atoms are in phase or maintain constant phase relationship
(b) different atoms emit photons in the same direction
(c) both a and b
(d) none

8. In spontaneous emissions, __________ .

(a) atoms are initially in the excited state
(b) transitions are without any aid of an external agency
(c) both a and b
(d) none

9. In conventional light sources, the ratio of spontaneous emission rate to stimulated emission rate is nearly _____ .

(a) 10^{10} (b) 10^{20} (c) 10^{5} (d) 10^{3}

10. In excited states, the atoms will remain for a time of __________ .

(a) 10^{8} s (b) 10^{-8} s (c) 10^{-3} s (d) 10^{-5} s

11. He–Ne gas laser is a __________ .

(a) continuous laser (b) pulsed laser
(c) both a and b (d) none

12. The ratio of the Helium and Neon gaseous atoms are __________ .

(a) 1:10 (b) 10:1 (c) 1:1 (d) 1:20

13. Ruby laser is a solid state __________ .

(a) pulsed, three-level laser (b) pulsed, four-level laser
(c) continuous, three-level laser (d) continuous, four-level laser

14. If the ruby rod contains 0.05 % of chromium atoms, then it appears in __________ colour.

(a) red (b) yellow (c) pink (d) green

15. At room temperatures, the threshold current density in heterostructure laser is of the order of __________ A/cm^2.

(a) 10^{5} (b) 10^{3} (c) 10^{2} (d) 10^{4}

16. In heterostructure strip geometry semiconductor lasers, the strip widths will vary from ___________ .

    (a) 5 to 30 μm (b) 50 to 100 μm
    (c) 5 to 150 μm (d) 1 to 5 μm

17. In buried hetero structure laser, the active region is completely surrounded by ___________ .

    (a) lower band gap and lower refractive index material
    (b) lower band gap and higher refractive index material
    (c) higher band gap and higher refractive index material
    (d) higher band gap and lower refractive index material

18. The cross-sectional area of the active region in buried heterostructure laser is as small as ___________ .

    (a) 50 μm^2 (b) 10 μm^2 (c) 1 μm^2 (d) 100 μm^2

19. Lasers are used in ___________ .

    (a) metal cutting and hole drilling (b) welding
    (c) surface treatment (d) all

20. The gas lasers give ___________ coherent beam compare to semiconductor laser.

    (a) less (b) equal (c) more (d) none

21. LASER stands for Light Amplification by ___________ of light.

    (a) stimulated emission (b) spontaneous emission
    (c) both a and b (d) none

22. Examples for ___________ emission light are glowing tube light, electric bulb, candle flame, etc.,

    (a) stimulated (b) spontaneous
    (c) both a and b (d) none

23. To form stimulated emission, a photon should make collision with an atom initially present in ___________ .

    (a) ground state (b) exited state
    (c) both a and b (d) none

24. He–Ne gas laser is a ___________ .

    (a) two-level laser (b) three-level laser
    (c) four-level laser (d) none of the above

25. Flat quartz plates are sealed at the ends of He–Ne gas laser to obtain ___________ .

    (a) polarized laser light (b) non-polarized laser light
    (c) polychromatic laser light (d) monochromatic laser light

26. He–Ne laser gives ___________ coloured laser light.

    (a) pink (b) red (c) green (d) orange

27. In ruby laser, chromium ions are responsible for stimulated emission of radiations whereas ———— ions are passive.

    (a) oxygen (b) aluminium (c) both a and b (d) none

28. Mostly, the wavelength of laser radiation from ruby laser is ___________.

    (a) 6943 Å (b) 6929 Å (c) 6328 Å (d) 1.15 nm

29. In homostructure semiconductor laser, the energy gap on one side of P-N junction is ___________ on the other side of the junction.

    (a) different as that (b) same as that
    (c) both a and b (d) none of the above

30. In heterostructure semiconductor laser, the energy gap on one side of the P-N junction is __________ on the other side of the junction.

(a) different as that (b) same as that (c) both a and b (d) none

31. At room temperature (300K), the threshold current density of homostructure laser is about __________ .

(a) 5.0×10^5 A/cm^2 (b) 5.0×10^4 A/cm^2
(c) 5.0×10^{-4} A/cm^2 (d) 5.0×10^{-5} A/cm^2

32. To reduce the operating currents to a large extent, heterostructure __________ geometry lasers are used.

(a) strip (b) rectangular (c) helical (d) parabolic

33. Basically, a ruby crystal is aluminium oxide doped with 0.05 to __________ % of chromium atoms.

(a) 5 (b) 10 (c) 15 (d) 0.5

34. Lasing action can be obtained with extremely small currents by using __________ structure lasers.

(a) homo (b) hetero
(c) buried homo (d) buried hetero

35. The threshold current in buried heterostructure is as low as __________.

(a) 5 mA (b) 10 mA (c) 15 mA (d) 15 A

36. Because of narrow angular spread, laser beam can be used for the communication between __________ and moon or other satellites.

(a) earth (b) sun (c) both a and b (d) none

37. Lasers are used in __________ the components of IC's.

(a) fabricating (b) trimming (c) both a and b (d) none

Answers

1. a	2. c	3. a	4. b	5. d	6. d	7. c	8. c	9. a	10. b	11. a	12. b
13. a	14. c	15. b	16. a	17. d	18. c	19. d	20. c	21. a	22. b	23. b	24. c
25. a	26. b	27. c	28. a	29. b	30. a	31. b	32. a	33. d	34. d	35. c	36. a
37. b											

Review Questions

1. Explain the principle, construction and working of a semiconductor laser. (Set-3–June 2005)
2. Explain the purpose of an active medium in a gas laser. (Set-3–Nov. 2004)
3. State the applications of lasers. (Set-1–Nov. 2003), (Set-2–Nov. 2003), (Set-3–Nov. 2003)
4. Derive the relation between the probabilities of spontaneous emission and stimulated emission in terms of Einstein's coefficients.
(Set-4–Sept. 2007), (Set-1, Set-3–May 2007), (Set-2–Nov. 2004)
5. Explain the characteristics of a laser.
(Set-1–May 2008), (Set-2, Set-3–Sept. 2007), (Set-2–Sept. 2006), (Set-1–Nov. 2004)

6. With the help of suitable diagrams, explain the principle, construction and working of a ruby laser. (Set-4–Nov. 2004)

7. What do you understand by population inversion? How it is achieved? (Set-4–Sept. 2007), (Set-1, Set-3–May 2007), (Set-2–Nov. 2004)

8. Mention any two applications of laser, each in the field of scientific research, engineering and medicine. (Set-2–June 2005), (Set-1–Nov. 2004), (Set-1–May 2003)

9. Explain the characteristics of laser beam. (Set-2, Set-3–Sept. 2007), (Set-2–Sept. 2006), (Set-2–June 2005), (Set-1–Nov. 2004), (Set-1–May 2003)

10. Explain the need of a cavity resonator in a laser, with the help of suitable diagrams explain the principle, construction and working of a ruby laser. (Set-1–Sept. 2006), (Set-4–May 2003)

11. Explain the purpose of an active medium in a gas laser. With the help of suitable diagrams, explain the principle, construction and working of a Helium–Neon laser. (Set-3–Nov. 2004), (Set-3–May 2003)

12. What do you understand by population inversion? How it is achieved? Derive the relation between the probabilities of spontaneous emission and stimulated emission in terms of Einstein's coefficients. (Set-2–May 2003)

13. With neat diagram, explain the construction and working of He–Ne gas laser. (Set-1–Nov. 2003)

14. Describe the construction and working of a ruby laser. (Set-1–May 2008), (Set-3–Sept. 2007), (Set-2–June 2005), (Set-1–Nov. 2004), (Set-1–May 2003)

15. Write the applications of lasers. (Set-4–Sept. 2006), (Set-4–May 2006), (Set-3–June 2005)

16. Describe the principle, construction and working of He–Ne laser. (Set-2–Nov. 2003)

17. Describe the principle, construction and working of a semiconductor laser. (Set-4–May 2006), (Set-3–Nov. 2003)

18. Distinguish between spontaneous emission and stimulated emission of the light. (Set-2, Set-3–May 2008), (Set-1–May 2006)

19. With the help of a suitable diagram, explain the principle, construction and working of a semiconductor laser. (Set-4–May 2008)

20. Write any four applications of laser. (Set-1–May 2008), (Set-2–Sept. 2008)

21. Distinguish between homo-junction semiconductor laser and hetero-junction semiconductor laser. (Set-2, Set-3–May 2008), (Set-4–Sept. 2008)

22. Describe the various methods to achieve population inversion relating to lasers. (Set-4–May 2008), (Set-3–Sept. 2008)

23. Explain the terms (i) absorption, (ii) spontaneous emission, (iii) stimulated emission, (iv) pumping mechanism, (v) population inversion and (vi) optical cavity. (Set-2–May 2007)

24. Mention the medical applications of laser. (Set-2–May 2007)

25. With neat sketch explain the construction and working of a ruby laser. (Set-2–Sept. 2007), (Set-2–Sept. 2006)

26. Explain the following typical characteristics of a laser (i) coherence, (ii) divergence and monochromaticity. (Set-3–May 2006), (Set-3–Sept. 2006)

27. Explain the principle and working of a ruby laser. (Set-3–May 2006), (Set-3–Sept. 2006)

28. What is population inversion. (Set-2–Sept. 2007), (Set-2–Sept. 2006)
29. With neat diagrams, describe the construction and action of ruby laser. (Set-4–Sept. 2006)
30. Explain the following (i) life time of an energy level, (ii) optical pumping process and (iii) metastable states. (Set-1–May 2006)
31. Discuss briefly the different methods of producing laser light. (Set-1–May 2006)
32. Explain with a neat diagram (i) absorption, (ii) spontaneous emission and (iii) stimulated emission of radiation. (Set-4–May 2007)
33. With necessary theory and energy level diagram, explain the working of a He-Ne gas laser. (Set-1–Sept. 2007), (Set-2–May 2006)
34. Mention some important applications of lasers. (Set-1–Sept. 2007), (Set-2–May 2006)
35. What is population inversion? How it is achieved by optical pumping. (Set-4–May 2007)
36. Discuss, how lasers are helpful in induced fusion and isotope separation process. (Set-3–Sept. 2007)
37. What is population inversion relating to laser action? Explain. (Set-1, Set-4–Sept. 2008)
38. Show that the ratio of Einstein's coefficient of spontaneous emission to Einstein's Coefficient of absorption is proportional to the cube of the frequency of the incident photon. (Set-1–Sept. 2008).
39. With the help of a suitable diagram, explain the principle, construction and working of a helium-neon laser. (Set-3–Sept. 2008)
40. Describe the construction and working of a semiconductor laser. (Set-2–Sept. 2008)
41. What are the important characteristics of laser radiation?
42. Explain the phenomenon of absorption, spontaneous and stimulated emission of radiation with two energy levels of an atom.
43. Explain the construction and working of a semiconductor laser.
44. Explain the various applications of lasers.
45. Write short notes on population inversion and Einstein's coefficients.
46. Describe semiconductor laser. Give the applications of lasers.
47. Explain the basic principle for producing laser beam. Write the medical applications of lasers.
48. What is population inversion in a laser? How it is achieved? What are the advantages of lasers in communication?
49. Describe He–Ne laser.
50. What are Einstein's coefficients?
51. Explain the terms: stimulated emission and population inversion. Mention the applications of lasers in the field of communication and medicine.
52. Explain in detail the working of a semiconductor laser.
53. Explain the principle and working of a semiconductor laser.
54. Write short notes on energy level diagram of He–Ne laser.
55. Write briefly on Einstein's coefficients.
56. Write short notes on stimulated emission.
57. Write short notes on semiconductor laser.

CHAPTER 7

Fibre Optics

7.1 Introduction

Optical fibre is a long, thin transparent dielectric material made up of glass or plastic, which carries electromagnetic waves of optical frequencies [visible to infrared] from one end of the fibre to the other by means of multiple total internal reflections. Thus, optical fibres work as wave guides in optical communication systems. An optical fibre consists of an inner cylindrical material made up of glass or plastic called core. The core is surrounded by a cylindrical shell of glass or plastic called the cladding. The refractive index of core (n_1) is slightly larger than the refractive index of cladding (n_2), [i.e., $n_1 > n_2$]. Typical refractive index values are $n_1 = 1.48$ and $n_2 = 1.46$. The core diameter is ≈ 50 μm and the thickness of cladding is $\approx$ 1 or 2 wavelengths of light propagate through the fibre. The cladding is enclosed in a polyurethane jacket as shown in Fig. 7.1. This layer protects the fibre from the surrounding atmosphere. Many fibres are grouped to form a cable. A cable may contain one to several hundred such fibres.

7.2 Principle of optical fibre, acceptance angle and acceptance cone

Principle: Once light rays enter into core, they propagate by means of multiple total internal reflections at the core-cladding interface, so that the rays travel from one end of the optical fibre to the other. The phenomenon of total internal reflection in a straight optical fibre is explained in the following way. Let the refractive index of the core is n_1 and that of the cladding is n_2 such that $n_1 > n_2$. As shown in Fig. 7.2, a ray of light *AO* is incident at 'O' on the end face of core; let this ray makes an angle of incidence θ_0 with the axis of the fibre. This ray is refracted into the core and passes along *OB*, the angle of refraction in the core is, say θ_1. The ray *OB* is incident on the core-cladding interface with an angle of incidence, $90°$-θ_1. Suppose this angle of incidence is equal to the critical angle [$\theta_c = 90° - \theta_1$] in core at the core-cladding interface, then the angle of refraction in cladding is 90°, so that the ray (*BC*) passes along the interface between core and cladding. If the angle of incidence for a ray at the end face is less than θ_0, then the angle of refraction is less than θ_1 and angle of incidence at the core-cladding interface is lager than critical angle, so the ray suffers total internal reflection at the

Figure 7.1 Optical fibre

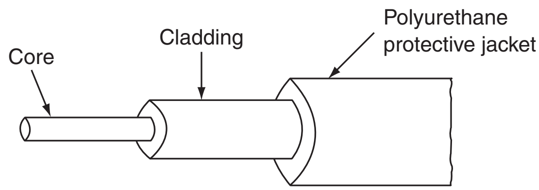
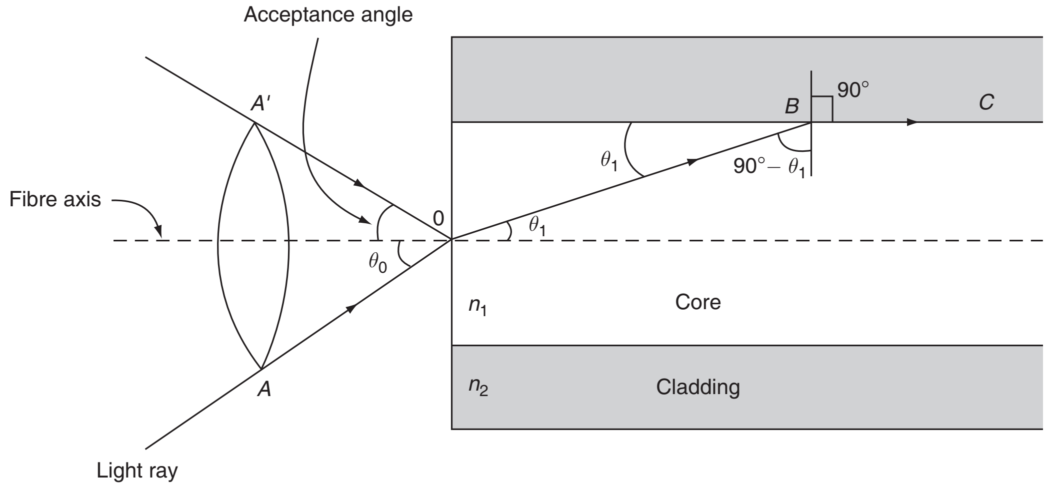


Figure 7.2 Light propagation in an optical fibre



core-cladding interface. If the angle of incidence for a ray at the end face is larger than θ_0, then the angle of refraction is larger than θ_1 and the angle of incidence at the core-cladding interface is less than critical angle so that the ray will be refracted into the cladding and get lost in it due to absorption.

So, all those rays which enter the core at an angle of incidence less than θ_0 will have refracting angles less than θ_1. As a result, their angles of incidence at the interface between core and cladding will be more than critical angle. As a consequence, they will be totally reflected in core and travel by multiple total internal reflections.

Acceptance angle and acceptance cone: As shown in Fig. 7.2, if the ray AO is rotated around the fibre axis keeping the angle of incidence θ_0 constant, it results in a conical surface. As such, only those rays which are within this cone suffer total internal reflections so that they confine to the core for propagation. If a ray falls at the end face of the optical fibre at an angle greater than θ_0 or out of the cone, that ray does not undergo total internal reflection at the core-cladding interface, it enters into cladding material and gets lost in the cladding material. Thus, for light rays to propagate through the optical fibre by total internal reflection, they must be incident on the fibre core within the angle θ_0. This angle is known as acceptance angle. Acceptance angle is defined as the maximum angle of incidence at the end face of an optical fibre for which the ray can be propagated in the optical fibre. This angle is also called acceptance cone half-angle.

A cone obtained by rotating a ray at the end face of an optical fibre, around the fibre axis with acceptance angle is known as acceptance cone. Expression for acceptance angle is obtained by applying Snell's law at points B and 0°.

Snell's law at 'B' is:

$$n_1 \sin(90° - \theta_1) = n_2 \sin 90°$$

$$n_1 \cos\theta_1 = n_2$$

$$\cos\theta_1 = \frac{n_2}{n_1}$$

or $$\sin\theta_1 = \sqrt{1-\cos^2\theta_1}$$

$$= \sqrt{1-\frac{n_2^2}{n_1^2}} \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.1)}$$

Snell's law at 'O'

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

or $$\sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.2)}$$

Substitute Equation (7.1) in (7.2)

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\frac{n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.3)}$$

As the fibre is in air.
So, the refractive index $n_0 = 1$
Equation (7.3) becomes:

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2} \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.4)}$$

This is the equation for acceptance angle.

7.3 Numerical aperture (NA)

Numerical aperture represents the light-gathering capacity of an optical fibre. Light-gathering capacity is proportional to the acceptance angle, θ_0. So, numerical aperture can be represented by the sine of acceptance angle of the fibre i.e., $\sin\theta_0$.

Expression for numerical aperture (*NA*): Expression for numerical aperture can be obtained by applying Snell's law at 'O' and at 'B' in Fig. 7.2. Let n_1, n_2 and n_0 be the refractive indices of core, cladding and the surrounding medium (air), respectively. Applying Snell's law at the point of entry of the ray [i.e., at 'O'],

We have:

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.5)}$$

At point 'B' on the core-cladding interface, the angle of incidence $= 90° - \theta_1$. Applying Snell's law at 'B', we have:

$$n_1 \sin(90° - \theta_1) = n_2 \sin 90°$$

$$n_1 \cos \theta_1 = n_2$$

$$\cos \theta_1 = \frac{n_2}{n_1} \qquad \text{(or)}$$

$$\sin \theta_1 = \sqrt{1 - \cos^2 \theta_1} = \sqrt{1 - \frac{n_2^2}{n_1^2}} \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.6)}$$

Substituting Equation (7.6) in (7.5), we have:

$$n_0 \sin \theta_0 = n_1 \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.7)}$$

If the surrounding medium of the fibre is air, then $n_0 = 1$.

So, $\quad \sin \theta_0 = \sqrt{n_1^2 - n_2^2}$

According to the definition for numerical aperture (*NA*),

$$NA = \sin \theta_0 = \sqrt{n_1^2 - n_2^2} \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.8)}$$

Let the fractional change in the refractive index (Δ) be the ratio between the difference in refractive indices of core and cladding to the refractive index of core.

i.e., $\quad \Delta = \frac{n_1 - n_2}{n_1} \quad$ __________ (7.9)

(or) $\quad n_1 - n_2 = \Delta n_1 \quad$ __________ (7.10)

Equation (7.10) can be written as:

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 - n_2)(n_1 + n_2)} \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.11)}$$

Substituting Equation (7.10) in (7.11), we have:

$$NA = \sqrt{\Delta n_1 (n_1 + n_2)}$$

Since $\quad n_1 \approx n_2; \quad$ so, $\quad n_1 + n_2 \approx 2n_1$

$$\therefore \; NA = \sqrt{2\Delta n_1^2} = n_1\sqrt{2\Delta} \quad \text{\_\_\_\_\_\_\_\_\_\_ (7.12)}$$

Numerical aperture can be increased by increasing 'Δ' and thus enhances the light-gathering capacity of the fibre. We cannot increase Δ to a very large value because it leads to intermodal dispersion, which causes signal distortion.

Condition for light propagation in the fibre: If θ_i is the angle of incidence of an incident ray at the end of optical fibre, then the ray will propagate if $\theta_i < \theta_0$

(or) $\quad \sin \theta_i < \sin \theta_0$

(or) $\sin \theta_i < \sqrt{n_1^2 - n_2^2}$

(or) $\sin \theta_i <$ NA is the condition for propagation of light within the fibre.

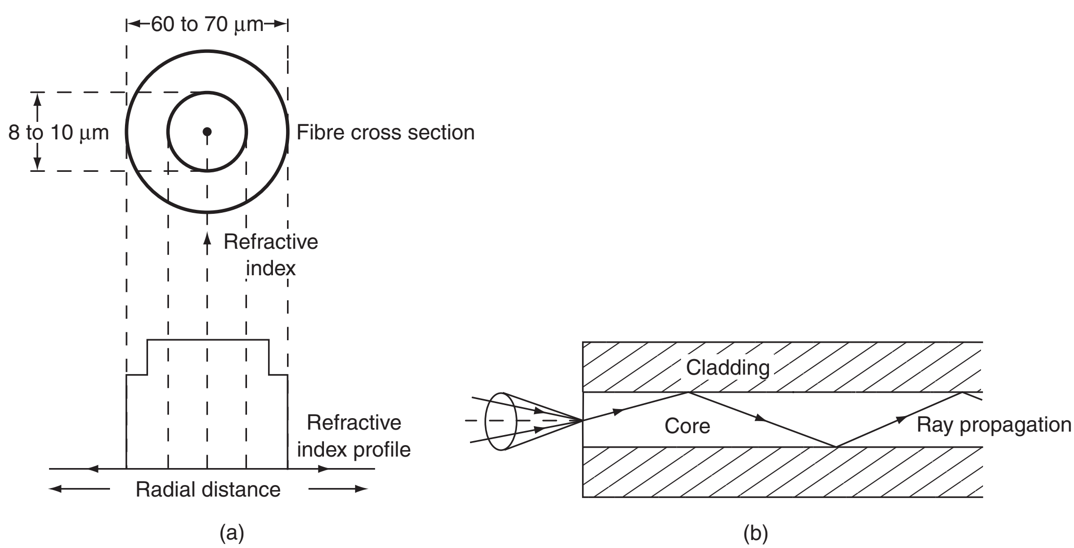
7.4 Step index fibres and graded index fibres—Transmission of signals in them

Based on the variation of refractive index of core, optical fibres are divided into: (1) step index and (2) graded index fibres. Again based on the mode of propagation, all these fibres are divided into: (1) single mode and multimode fibres. In all optical fibres, the refractive index of cladding material is uniform. Now, we will see the construction, refractive index of core and cladding with radial distance of fibre, ray propagation and applications of the above optical fibres.

(1) Step index fibre: The refractive index is uniform throughout the core of this fibre. As we go radially in this fibre, the refractive index undergoes a step change at the core-cladding interface. Based on the mode of propagation of light rays, step index fibres are of two types: (a) single mode step index fibres and (b) multimode step index fibres. Mode means, the number of paths available for light propagation in a fibre. We describe the different types of fibres below.

(a) Single mode step index fibre: The core diameter of this fibre is about 8 to 10 μm and outer diameter of cladding is 60 to 70 μm. There is only one path for ray propagation, so, it is called single mode fibre. The cross sectional view, refractive index profile and ray propagation are shown in Fig. 7.3. In this fibre, the transmission of light is by successive total internal reflections. i.e., it is a reflective type fibre. Nearly 80% of the fibres manufactured today in the world are single mode fibres. So, they are extensively used. Lasers are used as light source in these fibres. These fibres are mainly used in submarine cable system.

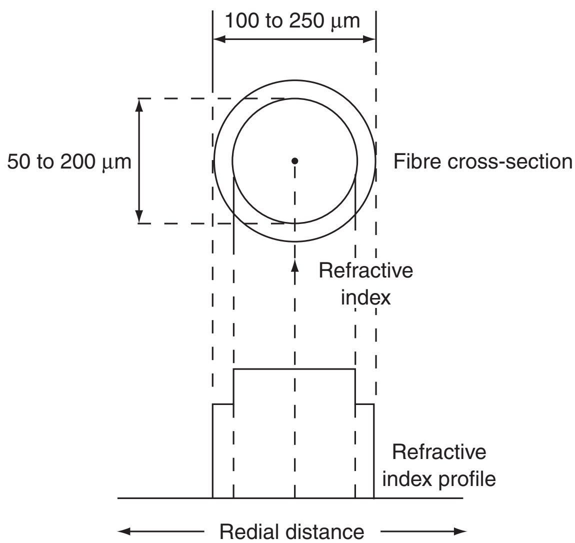
Figure 7.3 Single mode step index fibre: (a) Cross sectional view and refractive index profile and (b) ray propagation



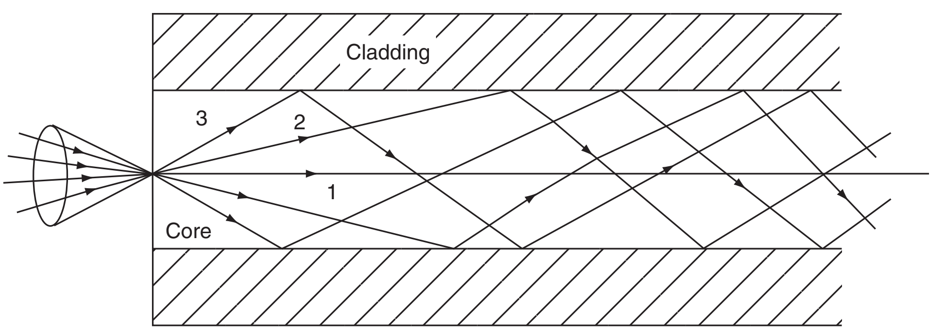
(b) Multimode step index fibre: The construction of multimode step index fibre is similar to single mode step index fibre except that its core and cladding diameters are much larger to have many paths for light propagation. The core diameter of this fibre varies from 50 to 200 μm and the outer diameter of cladding varies from 100 to 250 μm. The cross-sectional view, refractive index profile and ray propagation are shown in Fig. 7.4. Light propagation in this fibre is by multiple total internal reflections. i.e., it is a reflective type fibre. It is used in data links which have lower bandwidth requirements.

(c) Transmission of signal in step index fibre: Generally, the signal is transmitted through the fibre in digital form i.e., in the form of 1's and 0's. The propagation of pulses through multimode fibre is shown in Fig. 7.4(b). The pulse which travels along path 1 (straight) will reach first at the other end of fibre. Next, the pulse that travels along path 2 (zig-zag) reaches the other end with some time delay. Lastly, the pulse that travels along path 3 reaches the other end. Hence, the pulsed signal received at the other end is broadened. This is known as

Figure 7.4 Multimode step index fibre: (a) Cross sectional view and refractive index profile and (b) ray propagation



(a)



Ray propagation

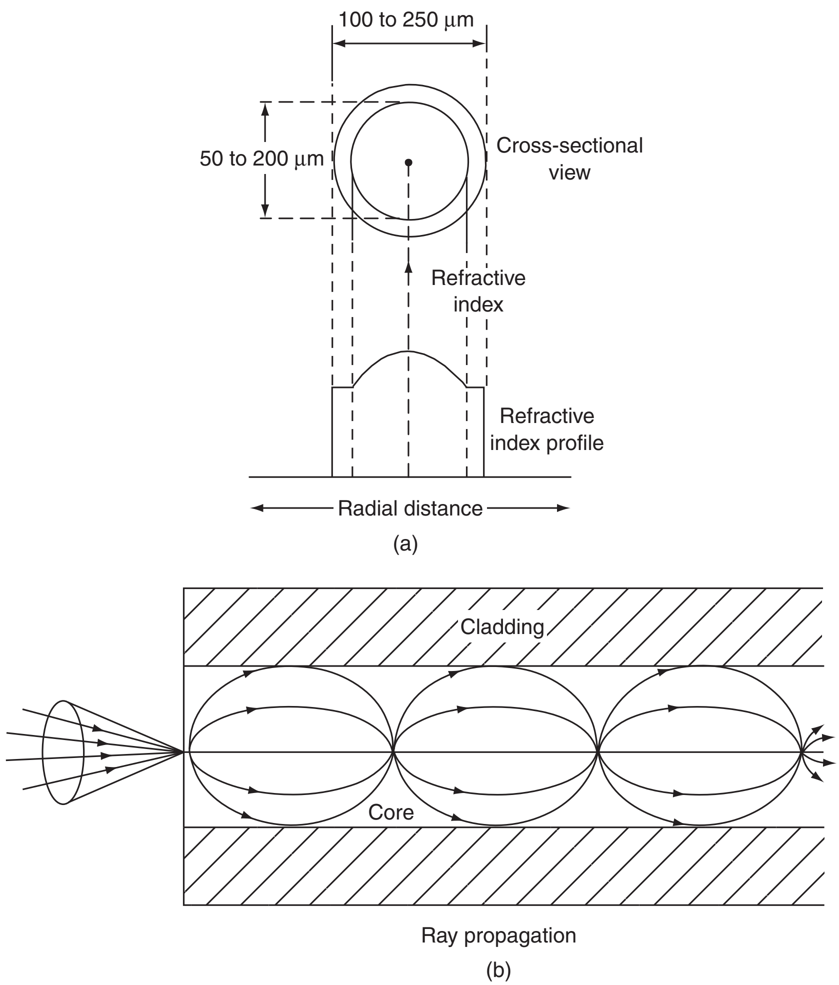
(b)

intermodal dispersion. This imposes limitation on the separation between pulses and reduces the transmission rate and capacity. To overcome this problem, graded index fibres are used.

(2) Graded index fibre: In this fibre, the refractive index decreases continuously from centre radially to the surface of the core. The refractive index is maximum at the centre and minimum at the surface of core. This fibre can be single mode or multimode fibre. The cross-sectional view, refractive index profile and ray propagation of multimode graded index fibre are shown in Fig. 7.5(a). The diameter of core varies from 50 to 200 μm and outer diameter of cladding varies from 100 to 250 μm.

The refractive index profile is circularly symmetric. As refractive index changes continuously radially in core, the light rays suffer continuous refraction in core. The propagation of light rays is not due to total internal reflection but by refraction as shown in Fig. 7.5(b). In graded index fibre, light rays travel at different speeds in different parts of the fibre. Near the surface of core, the refractive index is lower, so rays near the outer surface travel faster than the rays travel at the centre. Because of this, all the rays arrive at

Figure 7.5 Multimode graded index fibre: (a) Cross sectional view and refractive index profile and (b) ray propagation



the receiving end of the fibre approximately at the same time. This fibre is costly. Either laser or LED is used as light source. Its typical applications is in the telephone trunk between central offices.

Transmission of signal in graded index fibre: In multimode graded index fibre, large number of paths are available for light ray propagation. To discuss about intermodal dispersion, we consider ray path 1 along the axis of fibre as shown in Fig. 7.5(b) and another ray path 2. Along the axis of fibre, the refractive index of core is maximum, so the speed of ray along path 1 is less. Path 2 is sinusoidal and it is longer; along this path refractive index varies. The ray mostly travels in low refractive index region, so the ray 2 moves slightly faster. Hence, the pulses of signals that travel along path 1 and path 2 reach other end of fibre simultaneously. Thus, the problem of intermodal dispersion can be reduced to a large extent using graded index fibres.

7.5 Differences between step index fibres and graded index fibres

Step Index Fibre	Graded Index Fibre
1. The refractive index of the core is uniform and step or abrupt change in refractive index takes place at the interface of core and cladding in step index fibres.	1. The refractive index of core is non-uniform, the refractive index of core decreases parabolically from the axis of the fibre to its surface.
2. The light rays propagate in zig-zag manner inside the core. The rays travel in the fibre as meridional rays and they cross the fibre axis for every reflection.	2. The light rays propagate in the form of skew rays or helical rays. They will not cross the fibre axis.
3. Signal distortion is more in case of high-angle rays in multimode step index fibre. In single mode step index fibre, there is no distortion.	3. Signal distortion is very low even though the rays travel with different speeds inside the fibre.
4. The bandwidth is about 50 MHz km for multimode step index fibre whereas it is more than 1000 MHz Km in case of single mode step index fibre.	4. The bandwidth of the fibre lies in between 200 MHz Km to 600 MHz Km even though theoretically it has an infinite bandwidth.
5. Attenuation of light rays is more in multimode step index fibres but for single mode step index fibres, it is very less.	5. Attenuation of light rays is less in graded index fibres.
6. NA of multimode step index fibre is more whereas in single mode step index fibres, it is very less.	6. NA of graded index fibres is less.

7.6 Differences between single mode fibres and multimode fibres

Single Mode Fibres	Multimode Fibres
1. In single mode fibres there is only one path for ray propagation.	1. In multimode fibres, large number of paths are available for light ray propagation.
2. Single mode step index fibres have less core diameter (<10 μm) and the difference between the refractive indices of core and cladding is very small.	2. Multimode step index fibres have larger core diameter (50 to 200 μm) and the difference between the refractive indices of core and cladding is large.

3. In single mode fibres, there is no dispersion.	3. There is signal distortion and dispersion takes place in multimode fibres.
4. Signal transmission capacity is less but the single mode fibres are suitable for long distance communication.	4. Signal transmission capacity is more in multimode fibres. Because of large dispersion and attenuation, they are less suitable for long distance transmission.
5. Launching of light into single mode fibres is difficult.	5. Launching of light into multi mode fibres is easy.
6. Fabrication cost is very high.	6. Fabrication cost is less.
7. The V-number of a fibre $\left[V = \frac{2\pi}{\lambda} n_1 r \sqrt{2\Delta}\right]$ is less than 2.405 for single mode fibre. n_1, r are the refractive index and radius of core respectively, λ = wavelength of light that propagates through the fibre.	7. The V-number of a multimode fibre is greater than 2.405.

7.7 Attenuation in optical fibres

A very important parameter of an optical fibre is the attenuation of light signal in the fibre. Attenuation decreases light transmittance. Usually, the power of light at the output end of optical fibre is less than the power launched at the input end, then the signal is said to be attenuated. The signal attenuation is defined as the ratio of the input optical power (P_i) into the fibre to the power of light coming out at the output end (P_0). The attenuation coefficient is given as:

$$\alpha = \frac{10}{L} \log_{10} \frac{P_i}{P_0} \text{ dB/Km} \qquad \text{where } L \text{ is the length of the fibre}$$

The causes of attenuation are numerous, some of them are waveguide structure, material compositions, material dispersion, material scattering, microbending losses, mode coupling radiation losses, etc. The attenuation is the function of wavelength and material. Optical communication wavelengths are 0.8, 1.3 and 1.55 μm. The attenuation is mainly due to: (i) absorption and (ii) scattering.

(i) Absorption losses: In glass fibres, three different absorptions take place. They are ultraviolet absorption, infrared absorption and ion resonance absorption. Ion resonance absorption losses in pure fused silica are shown in Fig.7.6.

Absorption of uv radiation around 0.14 μm results in the ionization of valence electrons. Absorption of IR photons by atoms within the glass molecules causes heating. This gives absorption peak at 8 μm, also minor peaks at 3.2, 3.8 and 4.4 μm. The OH^- ions of water trapped during manufacturing causes absorption at 0.95, 1.25 and 1.39 μm as shown in Fig. 7.6. The presence of other impurities such as iron, copper and chromium also causes absorption. All these absorptions results in absorption loss in the fibre.

(ii) Scattering losses: The molten glass, when drawn into a very thin fibre under proper tension causes submicroscopic variation in the density of glass in the fibre takes place. The dopants added to glass to vary the refractive index also leads to inhomogenieties in the fibre. The microscopic variation of density and inhomogenieties acts as reflecting and refracting facets, these scatter a small portion of light passing through the glass. Thus, the scattering losses. If the size of density-fluctuating regions is of the order of $\lambda/10$ or less then they act as point source scattering centre. This kind of scattering is known as Rayleigh scattering. The scattering losses is proportional to $1/\lambda^4$. On this basis, the scattering losses at a wavelength of 1.3 μm is about 0.3 dB/Km whereas at a wavelength of 0.7 μm it is about 5 dB/Km. The Rayleigh scattering losses for silica is shown in Fig. 7.7.

Figure 7.6 Ion resonance absorption loss effects in fused silica glass fibres

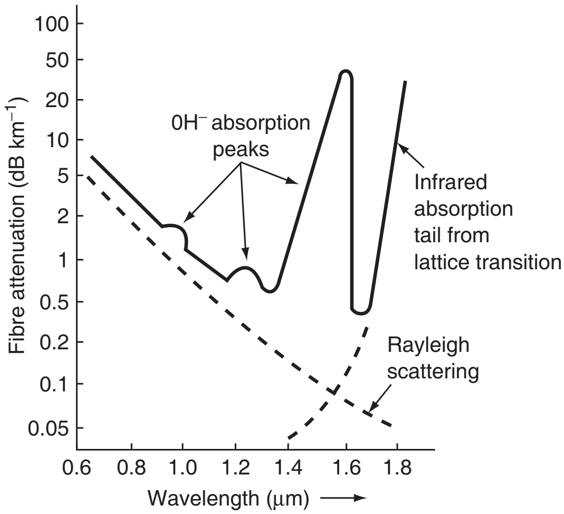
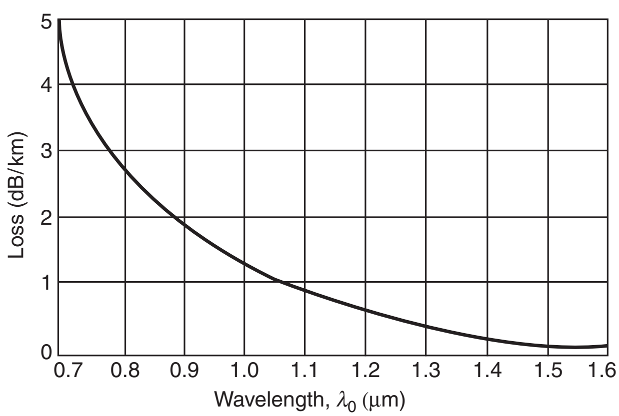


Figure 7.7 Rayleigh scattering losses in silica fibres



(iii) Bending Losses: In a bent fibre, there is loss in power of the transmitted signal called bending losses. Einstein explained the bending losses. According to Einstein's theory of relativity, the part of the ray that enters into cladding will travel faster. The energy associated with this part of the ray is lost. This loss can be represented by absorption coefficient (α)

$$\alpha = C \exp\left(\frac{-R}{R_C}\right) \quad \text{where } C \text{ is constant}$$

R = radius of curvature of fibre bend and $R_C = \dfrac{r}{(NA)^2}$

r = radius of the fibre. The bends with radius of curvature is of magnitude of the fibre radius gives rises to heavy losses.

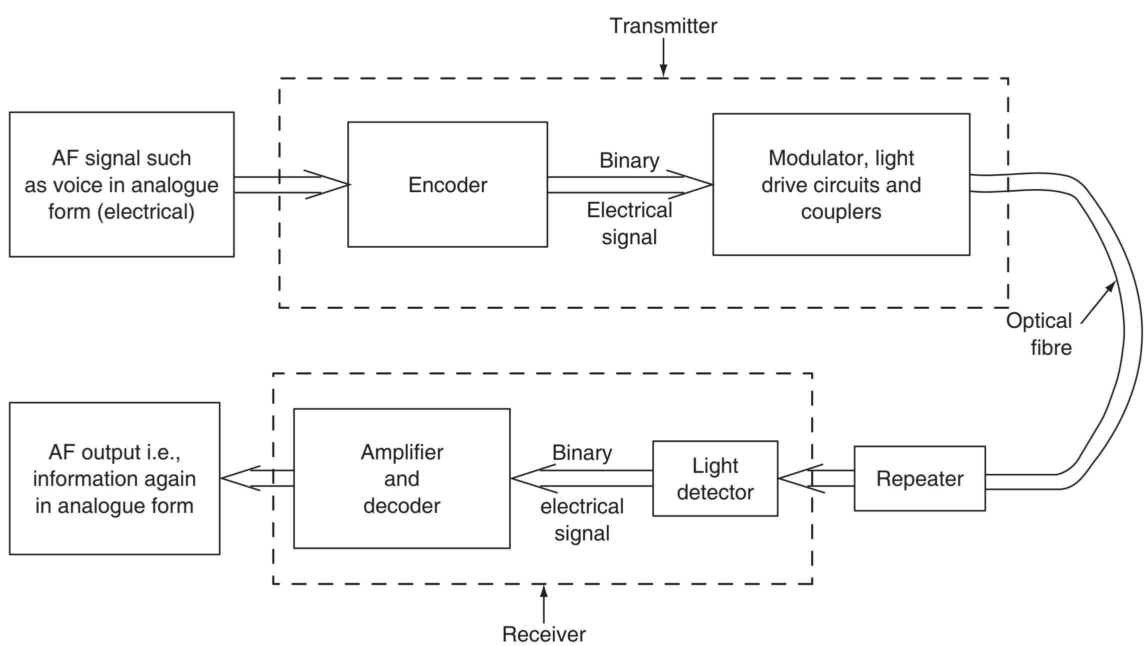
(iv) Microbending and wave guide losses: A large number of small bends present in the fibre causes large attenuation in the signal transmission. This is known as microbending loss. Usually, microbends are formed when an unsheathed fibre is wound in tension on a drum during manufacture. These bends will be more if the surface of drum is non-uniform.

During manufacturing, if proper care is not taken, then a continuous small variation in the fibre diameter or circularity is formed. This gives scattering loss, known as waveguide losses.

7.8 Optical fibres in communication

Fibre optics essentially deals with the communication [including voice signals, video signals or digital data] by transmission of light through optical fibres. Optical fibre communication system essentially consists of three parts: (a) transmitter (b) optical fibre and (c) receiver (Fig. 7.8). The transmitter includes modulator, encoder, light source, drive circuits and couplers. The light source can be a light emitting diode [LED] or a semiconductor laser diode. Basically, a fibre optic system simply converts an electrical signal [corresponds to analogue information such as voice] to binary data by an encoder and this binary data comes out as a stream of electrical pulses and these electrical pulses are converted into pulses of optical power by modulating the light emitted by the light source. That means the laser drive circuit directly modulates the intensity of the semiconductor

Figure 7.8 **Block diagram represents optical fibre communication system**



laser light with the encoded digital signal. This digital optical signal is launched into the optical fibre cable. The transmitter also has couplers to couple the transmitted light signals with the fibre. Fibres might require connectors to increase the length of the fibre medium. To transmit signals to long distances, repeaters are used after certain lengths in the optical fibre.

As the signal propagates in the fibre, it is subjected to attenuation and delay distortion. Attenuation is the loss of optical power due to absorption and scattering of photons. Even the leakage of light due to fibre bends adds to the attenuation effect. Delay distortion is because of spreading of pulses with time. The pulse spreading is mainly due to the variation in velocity of various spectral components of the pulse during its propagation in the fibre. When the attenuation and pulse spreading reaches beyond a limiting stage, then it may not be possible to retrieve the information from the light signal. Just at this threshold stage, a repeater is needed in the transmission path.

An optical repeater consists of a receiver and a transmitter arranged adjacently. The receiver section converts the optical signal into corresponding electrical signal, further this electrical signal is amplified and recast in the original form by means of an electrical regenerator i.e., reshape the signal and this signal is sent into an optical transmitter section, where the electrical signal is again converted back to optical signal and fed into an optical fibre.

Finally, at the end of optical fibre the signal is fed to the receiver. The receiver contains light detector. This can be either an Avalanche Photo Diode [APD] or a Positive Intrinsic Negative [PIN] diode. In the photodetector, the signal is converted in to pulses of electrical current, which is then fed to the decoder, which converts the sequence of binary data stream into an analogue signal as that fed at the transmitting end.

7.9 Advantages of optical fibres in communication

The following are the advantages of optical fibres in communication:

(i) Extremely wide band: The rate at which information can be transmitted is directly related to signal frequency. Light has very high frequency in the range of 10^{14} to 10^{15} Hz, as compared to radio frequencies $\sim 10^6$ Hz and microwave frequencies $10^8 - 10^{10}$ Hz. So, light can transmit information at a higher rate than systems that operate at radio frequencies or microwave frequencies.

(ii) Smaller diameter and light weight: Optical fibres are light-weight, smaller diameter and flexible; so, they can be handled more easily than copper cables.

(iii) Lack of cross-talk between parallel fibres: In copper cable communication circuits, signals often stray from one circuit to another, resulting in other calls being heard in the background.

This cross talk is negligible in optical fibres even when many fibres are cabled together.

(iv) Longer life-span: The life-span of optical fibres is expected to be 20−30 years as compared to copper cables, which have a life-span of 12−15 years.

(v) Temperature resistant: In contrast to copper cables, they have high tolerance to temperature extremes.

(vi) Easy maintenance: Optical cables are more reliable and easy to maintain than copper cables.

(vii) Much safer than copper cables: This is because only light and not electricity is being conducted.

(viii) Potential of delivering signals at low cost, because fibres are made up of silica, which is available in abundance in nature.

(ix) They possess low transmission loss and noise-free transmission is obtained as compared to copper cables. Since the transmission loss in optical fibres is as low as 0.2 dB/Km.

(x) Ruggedness and flexibility: Optical fibre cables are flexible, compact and extremely rugged.

7.10 Fibre optic sensing applications

Fibre optic sensors are used to monitor displacement, liquid level, flow, temperature and pressure, chemical composition etc. Optic fibre sensors can be divided into two types, they are:

(a) Intrinsic sensors/active sensors and (b) extrinsic sensors/passive sensors.

The Active sensors: In active sensors, the quantity to be measured acts directly on the fibre and modifies the radiation passing down the fibre.
The various active sensors are:

(i) Intensity modulated sensors: These are based on the change in refractive index, temperature, absorption, etc

(ii) Phase-modulated sensors: These involve the interference between the signal and reference in the interferometer. This leads to a shift in the interference fringes by the variable.

(iii) Polarization-modulated sensors: In this, a change in polarization state of the guided signal by the variable takes place.

(iv) Wavelength-modulated sensors: In this, the spectral dependent variation of absorption and emission by the variable takes place.

The passive sensors: In passive sensors, the modulation takes place outside the fibre. The fibre acts merely as a convenient transmission channel for light. The passive sensors has a sensor head and the sensed optical signal is transmitted to a remote point for signal processing. The table below gives the physical parameter to be measured using passive sensor and the modulation effects in the fibre.

Physical Quantity to be Measured	Modulation Effects in the Fibres
1. Temperature	Thermoluminescence
2. Pressure	Piezo optic effect
3. Density	Triboluminescence
4. Mechanical force	Stress birefringence
5. Electric field	Electro optic effect
6. Magnetic field	Magneto optic effect
7. Electric current	Electro luminescence
8. Nuclear radiation	Radiation-induced luminescence

Now, we study some sensors in detail.

(a) Displacement sensors

Intensity modulation of the transmitted light beam is utilized in this sensor. Figure 7.9 shows the displacement sensor.

Light from the source passes through one optical fibre and incident on the target. The reflected light reaches the detector through another optical fibre. Light reflected from the target and collected by the detector

Figure 7.9 **Displacement sensor**

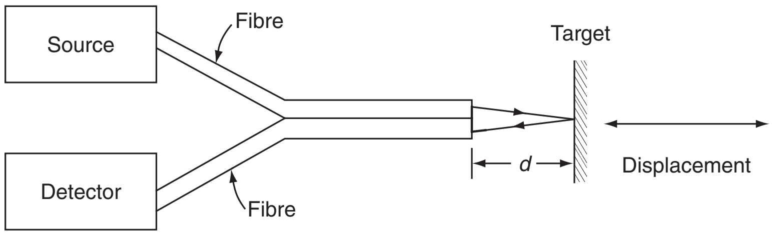
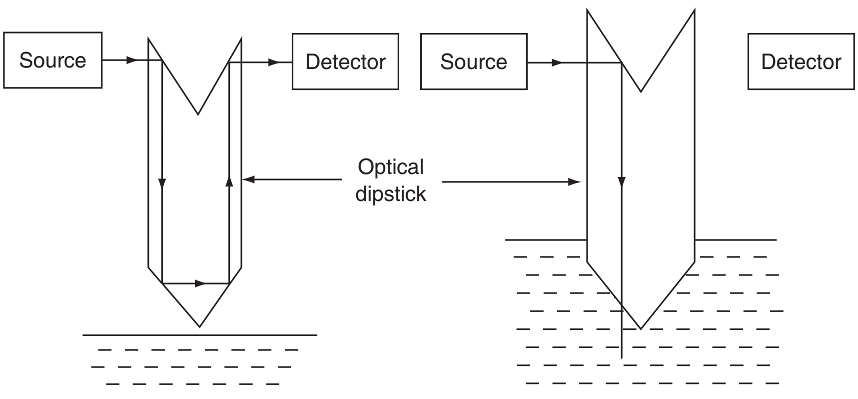


Figure 7.10 **Fluid level detector**



is a function of the distance between the fibre ends and the target. Hence, the position or displacement of the target may be registered at the optical detector. Further, the sensitivity of this sensor may be improved by placing the axis of the feed and return fibre at an angle to one another and to the target.

(b) Liquid level sensor

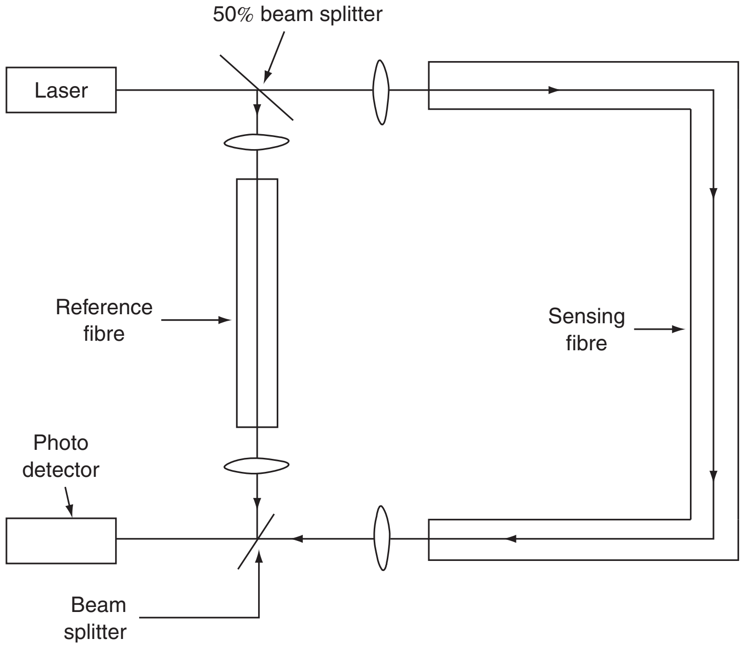
Figure 7.10 shows the operation of a simple optical fluid level switch. If the level of liquid is below the optical dipstick, due to total internal reflection, light from the source reaches the detector. If the level of liquid is above the chamfered end of the dipstick, then the light is transmitted into the fluid and the detector ceases to get light.

(c) Temperature and pressure sensor

When a single optical fibre is subjected to temperature or pressure variations, then its length and refractive index changes. This causes change in phase of light at the end of fibre. The change in phase of light is proportional to magnitude of the change in temperature or pressure. The phase changes can be measured by an interferometer method shown in Fig. 7.11.

Here, the light from a laser source is split into two beams of approximately equal amplitude by a 50% beam splitter. One beam is passed through sensing fibre, which is subjected to temperature or pressure variations and

Figure 7.11 Measurement of phase changes by interferometer method



the other beam through reference fibre, which is not subjected to any changes and is used for comparison. Light from these two fibres is superimposed using another beam splitter. Interference of these two waves gives fringes. The intensity of the fringe depends on the phase relation between the two waves. If the waves are in phase, then the intensity is maximum; this happens when the sensing fibre is not disturbed. The intensity is minimum if the waves are out of phase due to $\lambda/2$ change in length of sensing fibre. The intensity of interference fringes can be measured with a photodetector and temperature or pressure changes can be measured.

(d) Chemical sensors

Here, the sensing element is a modified fibre, and this sensing element senses the concentration of a chemical in terms of the phase change of the light wave. For example, in hydrogen sensor, palladium wire is fixed to the sensor. Hydrogen absorption changes the dimensions of the wire. This change produces strain in the optical fibre. This strain in the fibre changes the path length of light in the fibre. So, the concentration of hydrogen is proportional to the change in path length of light.

7.11 Applications of optical fibres in medical field

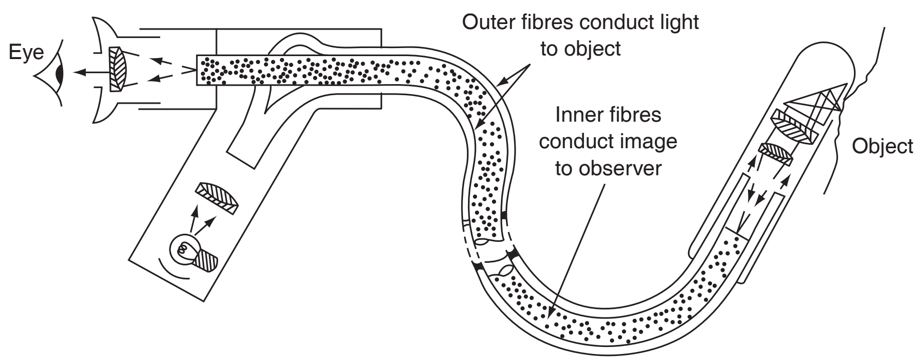
Optical fibre medical instruments may contain bundles of optical fibres. An optical fibre instrument used to see the internal parts of human body is endoscope. The endoscope facilitates the physicians to see the internal parts of body without performing surgery. The main part in endoscope is fibrescope. Based on application, the endoscopes are classified into:

(i) **Gastroscope** is used to examine the stomach. A gastroscope can be fitted with various parts to photograph tumours and ulcers. Laser-used gastroscope is used to remove objects that have been swallowed. Gastroscope can also guide a laser, used to destroy tumours.
(ii) **Bronchoscope** is used to see upper passages of lungs.
(iii) **Orthoscope** is used to see the small spaces within joints.
(iv) **Culdoscope** is used to test female pelvic organs.
(v) **Peritoneoscope** is used to test the abdominal cavity, lower parts of liver and gall bladder.

Also in ophthalmology, laser guided by the fibres is used to reattach the detached retina and to correct the defects in the vision.

The fabrication of fibrescope is used in endoscope. Fibrescope is shown in Fig. 7.12 below.

Figure 7.12 Flexible fibrescope



The fibrescope is also useful in industry. It could be used to examine welds, nozzles and combustion chambers inside the aircraft engines. These are not easily accessible for observation otherwise.

Formulae

1. $NA = \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$

2. $\Delta = \dfrac{n_1 - n_2}{n_1}$

3. $NA = n_1\sqrt{2\Delta}$

Solved Problems

1. *The refractive indices of core and cladding materials of a step index fibre are 1.48 and 1.45, respectively. Calculate: (i) numerical aperture, (ii) acceptance angle, and (iii) the critical angle at the core-cladding interface and (iv) fractional refractive indices change.*

(Set-1–May 2006)

Sol: Let the refractive index of core, $n_1 = 1.48$

and the refractive index of cladding , $n_2 = 1.45$

(i) Numerical aperture (NA) $= \sqrt{n_1^2 - n_2^2}$

$$= \sqrt{(1.48)^2 - (1.45)^2} = \sqrt{2.1904 - 2.1025} = \sqrt{0.0879} = 0.2965$$

(ii) Let θ_0 be the acceptance angle

Then, $\sin\theta_0 = \text{NA} = \sqrt{n_1^2 - n_2^2}$

$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2} = \sin^{-1}(0.2965) = 17°15'$

(iii) $n_2 \sin 90 = n_1 \sin\theta_c$ [θ_c = critical angle]

$\sin\theta_c = \dfrac{n_2}{n_1}$

$\theta_c = \sin^{-1}\dfrac{n_2}{n_1} = \sin^{-1}\left(\dfrac{1.45}{1.48}\right) = 78°26'$

(iv) The fractional refractive indices change, $\Delta = \dfrac{n_1 - n_2}{n_1} = \dfrac{1.48 - 1.45}{1.48} = 0.02$

2. *Calculate the angle of acceptance of a given optical fibre, if the refractive indices of the core and cladding are 1.563 and 1.498, respectively.*

(Set-3–Sept. 2008), (Set-1–May 2004)

Sol: Refractive index of core, $n_1 = 1.563$

Refractive index of cladding, $n_2 = 1.498$

Numerical aperture, $\text{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.563^2 - 1.498^2} = 0.446$

Acceptance angle, $\theta_0 = \sin^{-1}(\text{NA}) = \sin^{-1}(0.446) = 26°30'$.

3. *Calculate the fractional index change for a given optical fibre if the refractive indices of the core and cladding are 1.563 and 1.498, respectively.*

(Set-1–Sept. 2007), (Set-4–May 2004)

Sol: Refractive index of the core, $n_1 = 1.563$

Refractive index of cladding, $n_2 = 1.498$

The fractional refractive indices change, $\Delta = \dfrac{n_1 - n_2}{n_1} = \dfrac{1.563 - 1.498}{1.563} = 0.0416$.

4. *An optical fibre has a core material of refractive index 1.55 and cladding material of refractive index 1.50. The light is launched into it in air. Calculate its numerical aperture.*

(Set-4–May 2006), (Set-2–May 2004)

Sol: Refractive index of core, $n_1 = 1.55$

Refractive index of cladding, $n_2 = 1.50$

Numerical aperture, $\text{NA} = \sqrt{n_1^2 - n_2^2}$

$$= \sqrt{1.55^2 - 1.50^2} = 0.3905$$

5. *The numerical aperture of an optical fibre is 0.39. If the difference in the refractive indices of the material of its core and the cladding is 0.05, calculate the refractive index of material of the core.*

(Set-1–May 2008), (Set-3–May 2004)

Sol: Numerical aperture, NA = 0.39

The difference in refractive indices = $n_1 - n_2 = 0.05$ ____________ (1)

Refractive index of the core, $n_1 = ?$

From Equation (1)

$n_1 = n_2 + 0.05$ ____________ (2)

$$\mathrm{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 - n_2)(n_1 + n_2)}$$

$$0.39 = \sqrt{0.05 \times (n_1 + n_2)}$$

$$\frac{0.39^2}{0.05} = n_1 + n_2 = 3.042 \quad \text{____________ (3)}$$

Substituting Equation (2) in (3), we get:

$$3.042 = \mathrm{n}_2 + 0.05 + n_2 = 2n_2 + 0.05$$

$$n_2 = 1.496$$

$$\therefore\ n_1 = n_2 + 0.05 = 1.493 + 0.05 = 1.546.$$

6. *An optical fibre has a core material of refractive index 1.55 and cladding material of refractive index 1.50. The light is launched into it in air. Calculate its numerical aperture.*

(Set-4–May 2006), (Set-1–June 2005)

Sol: Refractive index of core, $n_1 = 1.55$

Refractive index of cladding, $n_2 = 1.50$

Numerical aperture, $\mathrm{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.55^2 - 1.50^2} = 0.3905$

7. *Calculate the numerical aperture and acceptance angle for an optical fibre with core and cladding refractive indices being 1.48 and 1.45, respectively.*

(Set-4–May 2007), (Set-4–June 2005)

Sol: Refractive index of core, $n_1 = 1.48$

Refractive index of cladding, $n_2 = 1.45$

Numerical aperture, NA = ?

acceptance angle, $\theta_0 = ?$

$$\mathrm{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.48^2 - 1.45^2} = 0.2965$$

$$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2} = \sin^{-1} 0.2965 = 17°15'.$$

8. *Calculate the refractive indices of core and cladding of an optical fibre with a numerical aperture of 0.33 and their fractional difference of refractive indices being 0.02.*

(Set-2–May 2006)

Sol: Refractive index of core, $n_1 = ?$

Refractive index of cladding, $n_2 = ?$

Numerical aperture, NA = 0.33

Fractional difference of refractive index, $\Delta = 0.02$

$$\Delta = \frac{n_1 - n_2}{n_1} \quad \text{or} \quad 0.02n_1 = n_1 - n_2$$

$n_2 = (1 - .02)$

$n_1 = 0.98n_1$

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$0.33 = \sqrt{n_1^2 - (0.98n_1)^2}$$

$0.33 = n_1 \times 0.198997$

$n_1 = 1.6583$

$n_2 = 0.98 \times 1.6583 = 1.625$

9. *An optical fibre has a numerical aperture of 0.20 and a cladding refractive index of 1.59. Find the acceptance angle for the fibre in water which has a refractive index of 1.33.*

(Set-3–May 2006), (Set-1, Set-2, Set-4–Sept. 2006), (Set-2–May 2007), (Set-2–Sept. 2007)

Sol: Numerical aperture of the fibre, NA = 0.20

Refractive index of cladding , $n_2 = 1.59$

Refractive index of water, $n_0 = 1.33$

Acceptance angle of fibre in water, $\theta_0 = ?$

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$\text{NA}^2 = n_1^2 - n_2^2$$

$$0.04 = n_1^2 - (1.59)^2$$

$$n_1^2 = 0.04 + (1.59)^2$$

$$= 2.5681$$

$$n_1 = 1.60253$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$= \frac{\sqrt{(1.60253)^2 - (1.59)^2}}{1.33}$$

$$= \frac{\sqrt{2.5681 - 2.5281}}{1.33} = \frac{0.2}{1.33}$$

$$= 0.15037$$

$$\theta_0 = \sin^{-1}[0.15037]$$

$$= 8°38' 56''$$

10. *A fibre has the core and cladding refractive indices 1.45 and 1.44 respectively. Find the relative refractive index difference.* (Set-4–Sept. 2007)

Sol: Refractive index of core (n_1) = 1.45

Refractive index of cladding (n_2) = 1.44

Relative refractive index difference (Δ)

$$= \frac{n_1 - n_2}{n_1} = \frac{1.45 - 1.44}{1.45} = 6.896 \times 10^{-3}$$

11. *The refractive index of core of step index fibre is 1.50 and the fractional change in refractive index is 4%. Estimate: (i) refractive index of cladding, (ii) numerical aperture, (iii) acceptance angle in air and (iv) the critical angle at the core–cladding interface.*

Sol: (i) The refractive index of the core, $n_1 = 1.50$

The fractional change in refractive index, $\Delta = \frac{n_1 - n_2}{n_1} = \frac{4}{100}$

where n_2 = refractive index of cladding

$$\therefore \frac{n_1 - n_2}{n_1} = 0.04$$

$$n_1 - n_2 = 0.04 \times 1.5 = 0.06$$

$$1.5 - n_2 = 0.06$$

$$\therefore n_2 = 1.44$$

(ii) Numerical aperture, $\text{NA} = \sqrt{n_1^2 - n_2^2}$

$$= \sqrt{(1.5)^2 - (1.44)^2} = \sqrt{2.25 - 2.0736} = \sqrt{0.1764} = 0.42$$

(iii) Acceptance angle, $\theta_0 = \sin^{-1}(\text{NA})$

$$= \sin^{-1}(0.42) = 24°50'$$

(iv) Critical angle, $\theta_c = \sin^{-1}\frac{n_2}{n_1}$

$$\sin^{-1}\frac{1.44}{1.50} = \sin^{-1} 0.96 = 73°44'$$

12. The refractive indices of core and cladding of a step index optical fibre are 1.563 and 1.498, respectively. Calculate: (i) numerical aperture and (ii) angle of acceptance in air.

Sol: Refractive index of core (n_1) = 1.563

Refractive index of cladding (n_2) = 1.498

(i) Numerical aperture (NA) = ?

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{1.563^2 - 1.498^2} = 0.446$$

(ii) Acceptance angle (θ_0) = ?

$$\theta_0 = \sin^{-1}(NA)$$

$$= \sin^{-1}(0.446)$$

$$= 26°30'$$

Multiple-choice Questions

1. The light sources used in fibre optic communication are ___________ .
   (a) LEDs (b) semiconductor lasers
   (c) phototransistors (d) both a and b

2. Acceptance angle is defined as the ___________ angle of incidence at the endface of an optical fibre, for which the ray can be propagated in the optical fibre.
   (a) maximum (b) minimum
   (c) Either a or b (d) none of the above

3. The core diameter of single mode step index fibre is about ___________ .
   (a) 60 to 70 μm (b) 8 to 10 μm
   (c) 100 to 250 μm (d) 50 to 200 μm

4. In multimode graded index fibre, light rays travel ___________ in different parts of the fibre.
   (a) at different speeds (b) with same speed
   (c) both a and b (d) none of the above

5. In optical communication system, the light detector is ___________ .
   (a) Avalanche Photo Diode (APD) (b) Positive Intrinsic Negative (PIN) diode
   (c) phototransistor (d) Either a or b

6. Optical fibres guides light waves by ___________ .
   (a) interference of waves (b) diffraction of waves
   (c) polarization of waves (d) by total internal reflection of waves

7. In an optical fibre, if n_1 and n_2 are the refractive indices of core and cladding, the condition for light propagation through fibre is __________ .

(a) $n_1 = n_2$ (b) $n_1 > n_2$
(c) $n_1 < n_2$ (d) none of the above

8. Loss of intensity of light in optical fibre is due to __________ .

(a) absorption (b) scattering
(c) reflection (d) both a and b

9. If n_1 and n_2 are the refractive indices of core and cladding, then numerical aperture (NA) of the fibre is ______ .

(a) $n_1^2 - n_2^2$ (b) $n_2^2 - n_1^2$
(c) $\sqrt{n_1^2 - n_2^2}$ (d) $\sqrt{n_1^2 - n_2^2}$

10. By increasing the refractive index of core material, the number of modes of propagation in an optical fibre __________ .

(a) increases (b) decreases
(c) remains same (d) none of the above

11. The life span of optical fibres is expected to be __________ .

(a) 40 to 50 years (b) about 100 years
(c) 20 to 30 years (d) less than 10 years

12. Fibre optic sensors are used to monitor __________ .

(a) displacement and flow (b) temperature
(b) pressure (d) all the above

13. Total internal reflection takes place when the angle of incidence is __________ the critical angle.

(a) greater than (b) less than (c) equal to (d) both a and b

14. Numerical aperture represents __________ capacity of a optical fibre.

(a) light gathering (b) light dissipation
(c) heat dissipation (d) magnetic lines gathering

15. In optical fibres, mode means __________ available for light rays to propagate in the fibre.

(a) the number of paths (b) the number of fibre in optical fibre cable
(c) the change in refractive index (d) none of the above

16. In multimode step index fibres, the core diameter is __________.

(a) 8 to 10 μm (b) 10 to 30 μm
(c) 50 to 200 μm (d) 100 to 250 μm

17. In multimode graded index fibre, the core refractive index profile is__________.

(a) circularly symmetric (b) non-linear
(c) step index (d) none of the above

18. The widely used optical fibre in the world is __________ .

(a) multimode step index fibre (b) multimode graded index
(c) single mode step index (d) none of the above

19. The acceptance angle is maximum if the critical angle is ___________.

(a) minimum (b) maximum (c) both a and b (d) none

20. In multimode optical fibre, the core diameter is ___________ in single mode fibre.

(a) lesser than (b) larger than (c) equal to (d) none

21. Optical fibres are made up with ___________ materials.

(a) semiconductors
(b) metals
(c) conductors
(d) dielectrics

22. In a reflective type optical fibre, the light rays pass from one end of the fibre to the other end by means of ___________.

(a) multiple total internal reflections
(b) refraction
(c) diffraction
(d) polarization

23. If the angle of incidence for a ray at the end face of an optical fibre is larger than acceptance angle, then the ray ___________.

(a) will not propagate in the fibre
(b) will propagate in the fibre
(c) both a and b
(d) none of the above

24. All the light rays which enter at a time into the multimode graded index fibre may arrive at ___________.

(a) different times at the other end of the fibre
(b) same time at the other end of the fibre
(c) both a and b
(d) none of the above

25. Delay distortion of light pulses in optical fibre is because of ___________ .

(a) spreading of pulses with time
(b) spreading of pulses with wavelength
(c) spreading of pulses with refractive index
(d) none of the above

26. Optical fibres carry very large information compared to copper cables because of ___________ .

(a) large thickness of fibre
(b) extremely wide bandwidth
(c) extremely less band width
(d) none

Answers

1. d	2. a	3. b	4. a	5. d	6. d	7. b	8. d	9. c	10. a
11. c	12. d	13. a	14. a	15. a	16. c	17. a	18. c	19. a	20. b
21. d	22. a	23. a	24. b	25. a	26. b				

Review Questions

1. Explain the advantages of optical fibres in communication. (Set-3–May 2004)
2. Explain the terms numerical aperture and acceptance angle.

(Set-4–May 2006), (Set-1–June 2005), (Set-2–May 2004)

3. Define acceptance angle and numerical aperture. Obtain an expression for numerical aperture of an optical fibre. (Set-4–May 2007), (Set-1–May 2006), (Set-4–June 2005)
4. What are the advantages of an optical fibre communication system over the conventional ones? (Set-4–Sept. 2007), (Set-4–Nov. 2003)
5. Describe the basic elements of a fibre optics communication system with a block diagram. (Set-4–Nov. 2003)
6. Write a note on the applications of optical fibres. (Set-1–Sept. 2007), (Set-4–May 2004)
7. Explain how the optical fibres are classified. (Set-3–Sept. 2008), (Set-1–May 2004)
8. Describe the construction of a typical optical fibre and give the dimensions of the various parts. (Set-4–May 2007), (Set-1–May 2006), (Set-4–June 2005)
9. With the help of a suitable diagram, explain the principle, construction and working of an optical fibre as a waveguide. (Set-4–May 2006), (Set-1–June 2005), (Set-2–May 2004)
10. Explain the principle of an optical fibre. (Set-3–Sept. 2008),(Set-1–May 2004)
11. Derive expressions for the numerical aperture and the fractional change of an optical fibre. (Set-1–Sept. 2007), (Set-3, Set-4–May 2004)
12. Describe the graded index optical fibres and explain the transmission of signal through it. (Set-3–Sept. 2007)
13. Derive an expression for the numerical aperture of an optical fibre. (Set-1–May 2008), (Set-3–Sept. 2006)
14. Explain the advantages of optical communication system. (Set-1–May 2008)
15. Derive the expressions for (i) acceptance angle and (ii) numerical aperture of an optical fibre. (Set-2–May 2008), (Set-4-Sept. 2008), (Set-3–Sept. 2006)
16. Describe different types of fibres by giving the refractive index profiles and propagation details. (Set-2–May 2008), (Set-4-Sept. 2008)
17. What are important features of optical fibres? (Set-3–May 2008)
18. Describe the communication process using optical fibres. (Set-3–May 2008)
19. Write the uses of fibre optics in different fields. (Set-3–May 2008)
20. Distinguish between light propagation in (i) step index optical fibre and graded index optical fibre. (Set-4–May 2008), (Set-2–May 2006)
21. Write a note on fibre optic medical endoscopy. (Set-4–May 2008)
22. Define the relative refractive index difference of an optical fibre. Show how it is related to numerical aperture. (Set-1, Set-3–May 2007)
23. Draw the block diagram of an optical fibre communication system and explain the function of each block. (Set-1, Set-3–May 2007)
24. Discuss the various advantages of communication with optical fibres over the conventional coaxial cables. (Set-2–May 2006)
25. Explain the principle behind the functioning of an optical fibre. (Set-2–Sept. 2007), (Set-2–May 2007), (Set-1, Set-4–Sept. 2006), (Set-3–May 2006)
26. Derive an expression for acceptance angle for an optical fibre. How it is related to numerical aperture? (Set-2–Sept. 2007), (Set-2–May 2007), (Set-1, Set-4–Sept. 2006), (Set-3–May 2006)
27. What is meant by an acceptance angle and numerical aperture; obtain mathematical expressions for acceptance angle and numerical aperture. (Set-2–Sept. 2006)

28. What is the principle of optical fibre communication? Explain. (Set-3–Sept. 2006)
29. Explain the basic principle of an optical fibre. (Set-3–Sept. 2007), (Set-1–Sept. 2008)
30. What are different losses in optical fibres? Write brief notes on each. (Set-3–Sept. 2007)
31. Explain the difference between a step index fibre and graded index fibre. (Set-4–Sept. 2007)
32. Write the applications of fibre optics in medicine and industry. (Set-1–Sept. 2008)
33. Describe the structure of an optical fibre. (Set-1–Sept. 2008)
34. Describe the step index fibre and explain the transmission of signal through it.
35. Write short notes on acceptance angle in a fibre.
36. Explain the propagation of light waves through an optical fibre.
37. Draw the block diagram of fibre optic communication system and explain the function of each element in the system.
38. Describe the structure of different types of optical fibres with ray paths.
39. Explain the terms: numerical aperture and acceptance angle of a fibre. Derive expressions for them.
40. Explain the transmission of signal in step index and graded index fibres.
41. Describe optical fibres in communication system.
42. What is the principle of optical fibre? Describe various types of optical fibres.
43. Distinguish between step index and graded index fibres with the help of refractive index profile.
44. What is mode in optical fibre? Distinguish between single mode and multimode step index fibres.
45. Describe the various fibre optic sensor applications.
46. Explain the advantages of optical fibre communications.
47. Write briefly on step and graded index optical fibres and numerical aperture of optical fibres.
48. Write briefly on numerical aperture of optical fibre, step and graded index optical fibres.
49. Write short notes on acceptance angle in optical fibres.
50. Write short notes on refractive index profiles of step-graded index fibres.

CHAPTER 8

Non-destructive Testing Using Ultrasonics

8.1 Introduction

The quality and soundness of welded, casted, forged, etc. materials or the components of turbines, internal combustion engines, parts in automobiles, locomotives, aeroplanes, rail road tracks, pipeline, riveted joints in boilers, plates, hulls of ships, etc. after manufacture in industries or in service can be determined by destructive tests or by non-destructive tests. By carrying destructive tests the components become spoiled and they become useless. And by using non-destructive tests the components or parts are not damaged. The main purpose of non-destructive testing (NDT) is to detect and identify the defects and flaws in metals and in products without spoiling them. Any deviation from perfection can be classified as a defect. NDT is a basic tool of industry in improving the product quality and maintaining quality levels. There is a large number of non-destructive testing methods. They are ultrasonics, X-ray radiography, gamma ray radiography, liquid penetration, magnetic particles testing, thermal methods, eddy current methods, etc. Of these large number of methods, we study ultrasonic methods of testing.

Ultrasonic method is widely used in industries to find the size, shape and location of flaws such as cracks, voids, laminations and inclusions of foreign materials, wall thickness of processed pipes and vessels. The *wall* thickness measurements are very important in corrosion studies. In ultrasonic method, sounds waves having frequencies between 200 KHz and 50 MHz are used.

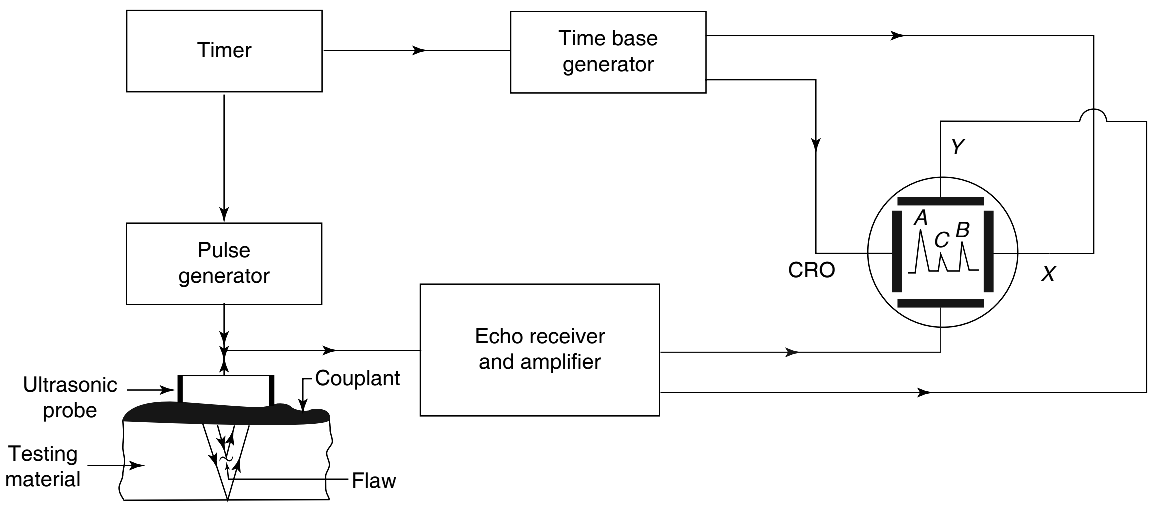
8.2 Principle of ultrasonic testing

Pulses of high-frequency ultrasonic waves generated by thin piezoelectric crystals present in ultrasonic probes are sent into the testing material through couplant. The ultrasonic pulses incident at the testing surface, rarer surface and the discontinuities or flaws inside the testing material (such as cracks, voids, inclusions of foreign materials, etc.) are reflected proportional to their sizes and these echoes are collected by the transducer. The time taken to receive the ultrasonic echoes from various surfaces and defects give the size, shape and location of defect and thickness of the testing material is the principle of ultrasonic testing.

8.3 Ultrasonic flaw detector

The ultrasonic flaw detector has several functional units. They have been shown in the block diagram in Fig. 8.1.

Figure 8.1 **Block diagram of an ultrasonic flaw detector**



The various functional units are described below:

(i) The timer controls the rate of generation of ultrasonic pulses, i.e., the pulse repetition frequency and synchronizes the signals generated from pulse generator and time base generator.
(ii) The pulse generator generates high-frequency alternating voltages of controlled amplitudes and passes them to ultrasonic transducer present in the ultrasonic probe.
(iii) The ultrasonic probe contains a piezoelectric crystal which acts as a transducer that converts the applied electrical voltage pulses into ultrasonic pulses of selected frequency. These pulses of ultrasonic waves are transmitted into the testing material through couplant. This effectively couple the ultrasonic waves from probe into the testing material by minimizing reflection losses. In immersion testing, the ultrasonic probe and the testing material are immersed in water or in liquid for uniform coupling. The ultrasonic pulses pass into the material and partial reflections take place at the contact face, rarer face and from defects of the material. These reflected ultrasonic pulses or echoes are received by the same transducer or another transducer. These echoes are converted into electrical signals by the transducer.
(iv) The echo receiver receives the electrical signals from the transducer and passes them to the amplifier for amplification of weak echo signals. After this they are sent to the *Y*-input plates of CRO.
(v) The time base generator supplies the sweep signals to the *X*-input plates of CRO. The time base generator controls the rate of sweep of the electron beam across the screen of the CRO. Using the time base generator, the length of the time base can be varied to a wide limit, i.e. one sweep of the screen may cater for any thickness of material, usually from few millimetres to several metres. The echo signals from the echo receiver are plotted in the CRO screen.

As shown in CRO screen, the initial strong echo pulse received by the receiver from the contact face of the material is indicated by the pulse *A*. The pulse *B* on the CRO screen is due to the reflection at the back wall of the testing material. If no flaw exists between the ultrasonic probe contact face and the opposite face of the testing material, then no peak appears between the peaks A and B on the CRO screen. Suppose if a small pulse or kink appears between the peaks A and B on the CRO screen then that corresponds to a flaw in the material. The peak 'C' in the CRO screen corresponds the presence of flaw in the material.

The thickness of the testing material can be estimated using the time gap between the pulses *A* and *B*, and the velocity of ultrasonic waves in the material. A pulse of ultrasonic wave travels from probe contact face to rarer face and again to the contact face, so that the distance travelled by the pulse is $2d$, where 'd' is the thickness of the material. If 't' is the time taken by the ultrasonic pulse to travel this distance, then

$$2d = V \times t$$

where V = velocity of the ultrasonic pulse in the material then $d = Vt/2$.

Thus, the thickness of testing material can be estimated. The time interval of echo pulse from initial contact face to flaw can give the depth of flaw from surface of the material.

8.4 Ultrasonic transducer

Certain non-centro symmetric crystalline materials show piezoelectric effect, i.e. by applying voltage across the opposite faces of the crystal the dimensions of the crystal will change. Also by applying mechanical pressure on two opposite faces of the piezoelectric crystal, opposite electrical charges are induced on the other two faces of the crystal and hence the potential between the faces. This voltage produced is proportional to the amount of strain. The crystalline materials such as quartz, rochelle salt, potassium dihydrogen phosphate (KDP), ammonium dihydrogen phosphate (ADP), lithium sulphate, tourmaline, etc. show piezoelectric effect. The main part of ultrasonic transducer is piezoelectric crystal and the ultrasonic probe uses this transducer. By applying alternating voltage across the thickness of a piezoelectric transducer, it can expand and contract continuously and hence it generates a compression wave normal to the surface of crystal in a surrounding medium. The frequency of vibration depends on its dimensions. As the thickness of crystal is reduced, the natural frequency of vibration of crystal increases.

In some ultrasonic testing techniques, two transducers are used: one transducer to transmit the ultrasonic beam and the other to receive the reflected sound waves (echoes). In many cases, single piezoelectric transducer is used as transmitter and as receiver. Ultrasonic waves are transmitted in the form of a series of pulses of extremely short duration and during the time interval between the transmissions the crystal can detect the echo signals, i.e., an ultrasonic wave is generated by exciting a piezoelectric crystalline material with a high-amplitude, transient electrical pulse from a high-voltage, high-current pulser, then a short burst of ultrasonic energy from the crystal is emitted. These pulses of energy travel through the material as stress waves and are reflected back by the far boundaries of the material or flaws in the material, these reflected pulses are received by the crystal. These stress pulses are converted into electrical signals and are displayed upon the face of CRO, from which the position of the flaw can be determined.

The magnitude of the piezoelectric effect in many crystals depends on the orientation of the applied force or electric field with respect to the crystallographic axes of the material. Piezoelectric elements are cut from the crystals and are identified by the axis perpendicular to the largest face of the cut. Thus a slab of crystal cut with its major faces perpendicular to the X-crystallographic axis is X-cut crystal. X-cut slab with edges

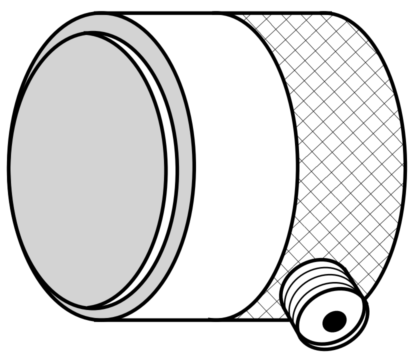
parallel to *Y*- and *Z*-axes is represented as 0° X-cut plate. If the edges are at an angle of 45° with respect to the *Y*- and *Z*-axes, then it is represented as 45° X-cut plate.

The piezoelectric effect is high along 0° X-cut and 45° X-cut of rochelle salt, the 0° and 45°-cuts of ADP. The 0° X-cut quartz is commonly employed for transducers. The 0° Y-cut of lithium sulphate is also used. Polycrystalline titanate elements and quartz crystal can be applied for the generation of high ultrasonic waves. Circular plates of piezoelectric crystal are used for ease of mounting and for greater resistance to edge chipping. To impress electric charge evenly the appropriate face of quartz is covered with a thin conductive film, usually a thin metal foil.

There are a variety of ultrasonic transducer probes. Mainly they are (i) contact transducers and (ii) immersion transducers. These are described below.

(i) Contact transducers: These are hand used direct contact inspection transducers. They are protected in a rugged casing to withstand sliding contact. They are designed to have easy gripe and move along a surface. They are provided with replaceable wear plates to increase its life time. This is shown in Fig. 8.2.

Figure 8.2 Contact transducer



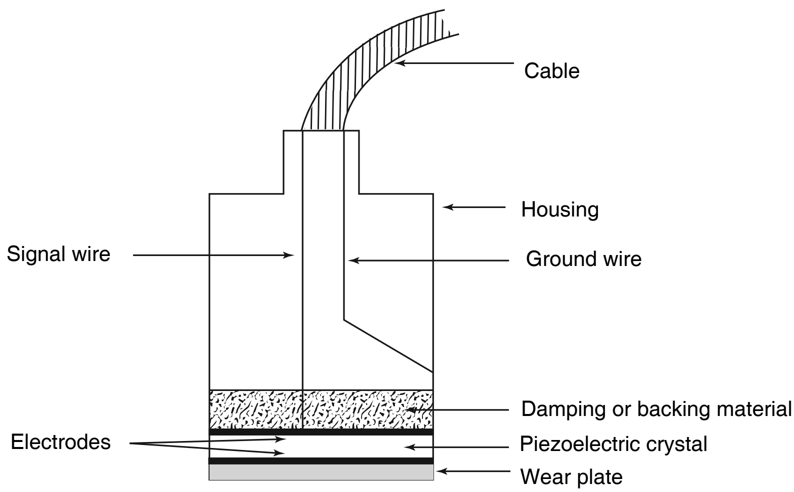
(ii) Immersion transducers: These transducers are operated in liquids. In these transducers, all connections are water tight. They do not make contact with the testing material. In immersion testing more sound energy is transmitted into the testing material because of the liquid couplant in between transducer and the surface of the testing material. Immersion transducers are planar, cylindrically focused or spherically focused lens at the bottom. This focusing improves the sensitivity and axial resolution.

For different applications, different types of transducers are constructed. They are (i) normal incidence shear wave transducers, (ii) angle beam transducers, (iii) paint brush transducers, (iv) delay time transducers and (v) dual element transducers.

Transducer construction

The construction of normal probe transducer is shown in Fig. 8.3. It consists of a piezoelectric crystal with electrodes on its two faces and housed in a cylindrical case. Small wire leads are attached to electrode surfaces by welding, soldering or cementing. The applied voltage pulses make the crystal to vibrate at its natural frequency. After step voltage, to die the oscillations quickly the crystal is backed with a damping material. The backed material is a fibrous or cellular plastic material.

Figure 8.3 Ultrasonic transducer probe



To get maximum energy out of the transducer, an impedance matching is placed between crystal and the face of the transducer. At the bottom of probe, the crystal is protected by a layer of metal or ceramic or perspex. Acoustic lenses can also be attached to the front surface of the transducer to act as a lens for focusing sound beam.

The effectiveness of the transducer depends on the quality factor Q, band width, frequency, sensitivity, acoustic impedance and resolving power. The Q factor of the transducer is

$$Q = \frac{F_r}{F_2 - F_1}$$

where F_r = resonant frequency of the crystal,
$F_2 - F_1$ = band width,
F_2 = frequency above F_r and
F_1 = frequency below F_r.

Usually, the Q value will vary from 1 to 10.

The sensitivity of the transducer is the ability of it to detect reflections or echoes from small defects or flaws. Sensitivity is proportional to the product of its efficiency as a transmitter and efficiency as a receiver. The acoustic impedance of a transducer is the product of its density and velocity of sound within it. The resolving power depends on crystal damping and band width.

8.5 Couplant

A portion of the incident ultrasonic waves will be reflected at the interface and a portion will be transmitted across the interface. Suppose Z_1 and Z_2 are the acoustic impedances of two media then for normal incidence the reflection coefficient is $R = (Z_2 - Z_1)/(Z_2 + Z_1)$ and the transmission coefficient is $T = 2Z_1/(Z_2 + Z_1)$, where $Z = \rho V_c$, ρ is the density of the material and V_c is the velocity of the compression wave in the medium. The reflection of sound at air and metal interface is nearly 100% for the normally used frequencies of ultrasonic testing. So ultrasonic waves cannot be transmitted easily from

air into metal. If a fluid such as oil or water is used as a couplant between the transducer crystal and the testing metal, then the reflection coefficient is reduced. By increasing the thickness of fluid coupling the transmission of ultrasonic waves will be improved.

The couplant that have been used in ultrasonic testing is usually a liquid material, which has been placed between the ultrasonic transducer and the testing material surface as shown in Fig. 8.4. The couplant makes that more ultrasonic energy to transmit from the transducer into the testing material by reducing the acoustic impedance mismatch. In contact ultrasonic probe testing, a thin film of oil, glycerin or water is usually used between the transducer and the surface of the test material. Usually immersion ultrasonic testing has been carried in a tank of water or in liquid, where the test material and partially the transducer are immersed in the water as shown in Fig. 8.5.

The surfaces on which the transducer is to be placed and moved should be smooth upto a certain extent, and a couplant must be placed between the surface of the testing material and the ultrasonic probe. Immersion testing has been carried out for inspection of slabs that have to be machined into air craft structural components, gas turbines, air craft wheels, etc. The screen display of immersion text will show a blip corresponding to the water–metal interface.

Figure 8.4 Use of couplant

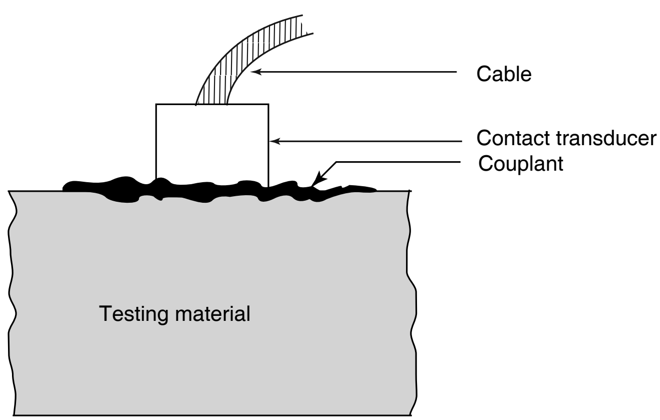
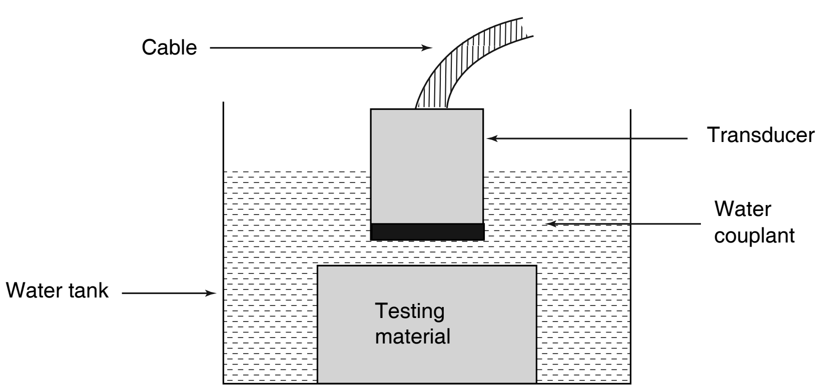


Figure 8.5 Immersion ultrasonic testing

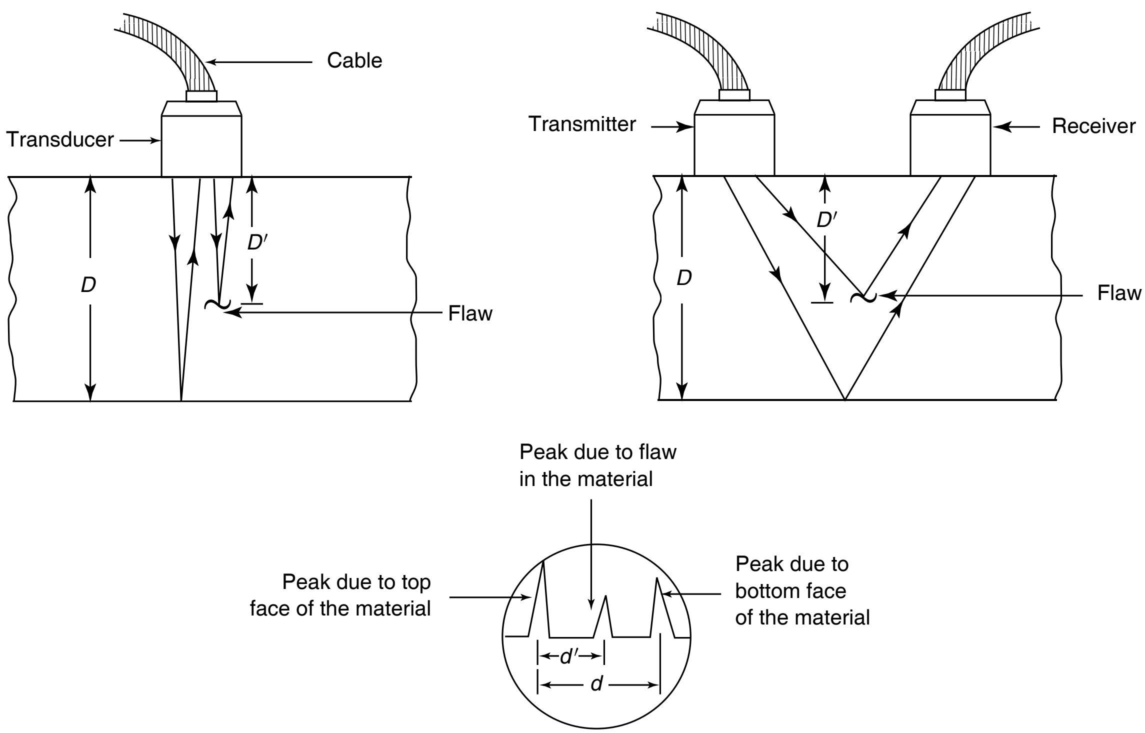


8.6 Inspection methods—Pulse echo testing technique

There are different techniques of ultrasonic inspection of materials. They are: (i) pulse echo, (ii) transmission, (iii) resonance, (iv) frequency modulation and acoustic image. Of these techniques, the pulse echo testing has been described below.

Pulse echo testing: In the ultrasonic pulse echo technique, pulses of ultrasonic beam from the ultrasonic transmitting transducer are passed normally or at a certain angle into the testing material through a couplant. The pulses are partially or completely reflected at the boundaries of discontinuities or flaws, at the front and back faces of the testing material. The reflected pulses are received either by the same transducer as shown in Fig. 8.6(a) in case of normal incidence or by another transducer called receiving transducer as shown in Fig. 8.6(b) for angle incidence. The pulses are received during a period when the transmitter is not driven by the same transducer or other receiving transducer.

Figure 8.6 (a) Single transducer pulse echo technique, (b) two transducer pulse echo technique and (c) CRO display with linear time base [$d':d = D':D$]



The received ultrasonic pulses are converted into electrical signals and they are displayed upon the face of CRO. From the CRO display shown in Fig. 8.6(c), the positions of peaks on the time axis indicate the depth of the defects or flaws inside the material. This is a convenient method, because the material can be tested from its one side only. Using this method, voids, cracks and non-metallic inclusions in metals can be

inspected. The sensitivity of this method is large, this is a widely used method, because the thickness of steel plates in the range of 1–200 mm can be measured easily. The thicknesses of ships hulls, oil tanks, pipes, etc. are measured. The testing is also carried by immersing the testing material in a tank of water. One angle probe can be used in the reflection mode to find defects as shown in Fig. 8.7. In this method, the flaw detector is accurately calibrated using a reference test block.

To detect surface defects, surface waves or Rayleigh waves are used as shown in Fig. 8.8. The wave gets reflected at the surface defect and the echo is detected by the same transducer.

Figure 8.7 Angle probe reflective pulse echo testing

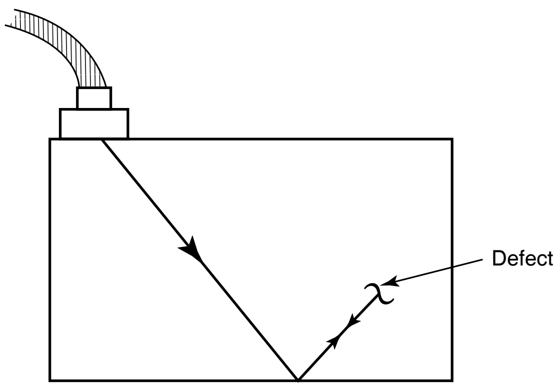
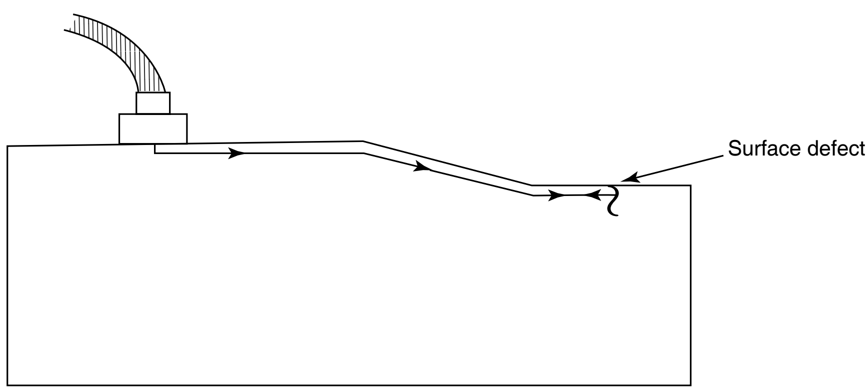


Figure 8.8 Surface defect pulse echo inspection

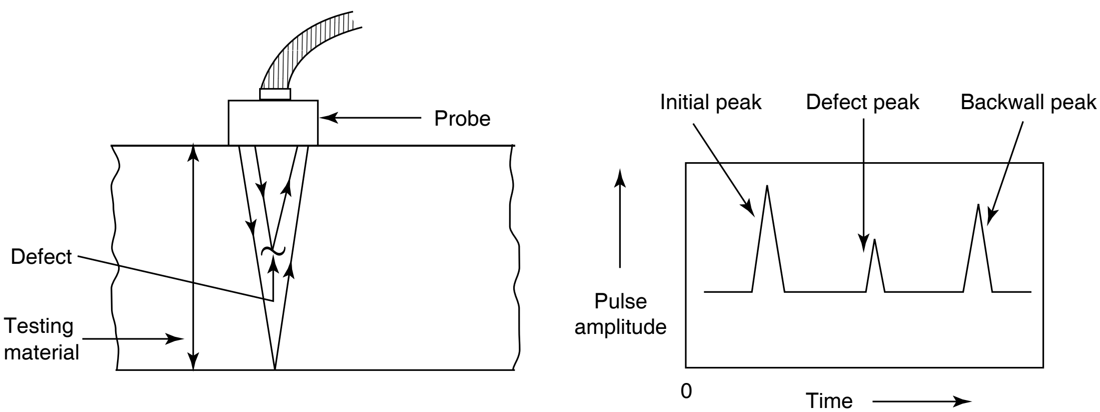


8.7 Different types of scans

A material can be scanned using ultrasonic testing and the data obtained is presented in different ways. These presentations facilitate different ways of looking the region of a material under test. Mainly we have three different types of scans. They are named as A-, B- and C-scans. These are described below.

A-scan: This shows a one-dimensional presentation of scanned data. This presents the existence of flaws or defects under the ultrasonic probe in the testing material. The A-scan shows the size and the position of defect under the probe. The A-scan presentation is shown in Fig. 8.9. The probe sends pulses of ultrasonic energy into the material.

Figure 8.9 A-scan display: (a) ultrasonic pulse echo testing of material and (b) representation of CRO screen display



From this a certain amount of ultrasonic energy is reflected at the top and bottom faces of the material. The echoes from the top and bottom faces give the initial and back wall peaks on the CRO screen. Suppose a defect is present under the probe in the material, which gives a small echo. This corresponds to a defect peak on the surface of CRO screen. The size of the defect is proportional to the height of the defect peak and the depth of the defect from the top of the material is proportional to the position of the defect peak on the horizontal sweep of CRO.

B-scan: In this scan, the ultrasonic probe is moved along a line on the top face of the material. This scan shows the reflections of ultrasonic pulse energy from the top and bottom faces of the testing material, also from the defects as the ultrasonic probe moves along a line on the surface of the material. The B-scan presents the cross-sectional view of the material along a line, and provides the depth of flaw from the surface of the material and also the length of flaw along the line of moment of the probe. The B-scan presentation is shown in Fig. 8.10. Here along the Y-axis the echo receiving time is taken and along the X-axis the scan distance along a line from a point on the surface of the testing material is taken. The line P indicates the reflection of ultrasonic pulse from the top surface and the line R is due to the reflection from the bottom surface of the testing material. The line Q is due to the flaw in the material.

C-scan: In this scan, the ultrasonic probe is sweeped continuously line by line on the complete top face of the material as shown in Fig. 8.11. This scan produce a plan view of the material and this is similar to radiograph. The C-scan display is confined to a computer-controlled immersion scanning system provided with an automated data acquisition system. A display memory stores the echo amplitudes obtained at various positions of the scan surface. The stored information is regenerated on CRO screen. The display shows the graphical representation of image of flaw as seen from the top view of the test material. The C-scan top and bottom surface reflections are not used; only the reflections from the flaws are used.

Figure 8.10 B-scan display

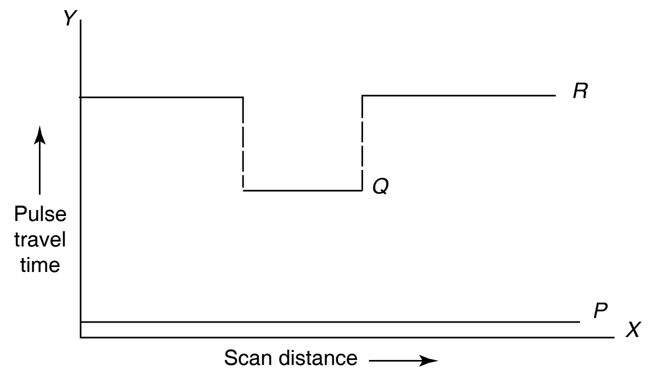
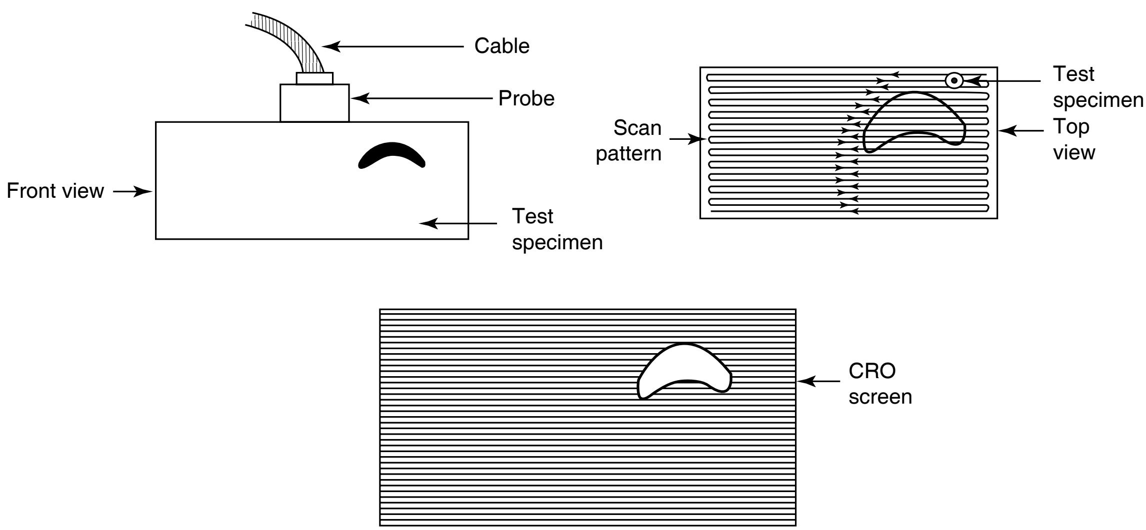


Figure 8.11 C-scan display

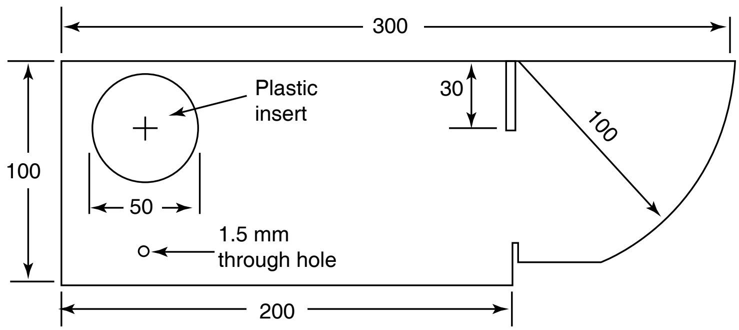


8.8 Inspection standards [Reference standards or calibration blocks]

The inspection standards are used for the precision and accuracy of measurement equipment. The inspection standards are used to establish consistency in measurements. The inspection standards help the inspector to estimate the size of flaws. In pulse echo inspection, the signal strength depends on the size of flaw and the distance of flaw from the transducer. The inspector can use inspection standard blocks in which he artificially creates a flaw of known size at nearly the same distance from the transducer to produce a signal. The flaw size can be estimated by comparing the signal received from the inspection standard and that received from actual flaw. The specially designed standards are required for many applications. The inspection standards are available in many shapes and sizes. The type of standard depends on the form and shape of the testing material.

The holes drilled and the notches in the inspection standards do not closely represent real flaws. The artificially produced defects are the better reflectors of sound energy and shows indication of similar sized flaws in the testing material. The International Institute of Welding (IIW) calibration block is shown in Fig. 8.12. There are three types of standard blocks for calibration of test equipment. They are (i) distance amplitude, (ii) area amplitude and (iii) test blocks as recommended by IIW.

Figure 8.12 IIW inspection standard block (with dimensions)



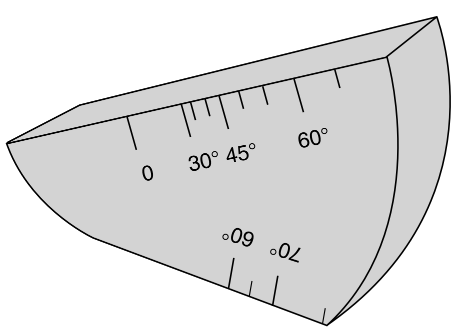
(i) Distance–amplitude blocks: These are used to determine the relationship between the depth and the amplitude for direct beam inspection of a given material. They are cylindrical in shape and have the same material as that of test piece. The bottom is flat and contains holes of specific size and of different lengths.

(ii) Area–amplitude blocks: Here the amplitude of an echo received by the transducer from flat bottom hole is proportional to the area of the bottom of the block. The blocks are made up of a material having the same acoustic properties as that of test component.

(iii) Calibration blocks: For the calibration of angle beam and direct beam probes, IIW steel calibration blocks are universally used. These blocks include notches, circular grooves and scales. These blocks are used to calibrate angle beam and normal beam incident inspections. The angle beam calibration block is shown in Fig. 8.13. This block was designed for the US Air Force in the field for instrument calibration, and it is used to make metal distance and sensitivity calibration in case of angle beam and normal beam inspections.

The miniature resolution block is used to find the near surface resolution, sensitivity of normal beam set up and to calibrate high-resolution thickness gauges.

Figure 8.13 Angle beam calibration block



Step and tapered calibration wedges are a variety sizes and configurations. Usually step wedges contain four or five steps and the tapered wedges with constant taper are used.

Distance/area amplitude correction blocks are a set of ten blocks. They are manufactured with aluminium or steel or with titanium.

Each block contains a single flat bottomed and a plugged hole. Aluminium sets are manufactured as per the requirements of ASTM 127 and steel sets are as per ASTM 428 [ASTM is an abbreviation for American Society for Testing of Metals]. The hole sizes and metal path distances are given below.

- 3/64″ at 3″
- 5/64″ at 1/8″, 1/4″, 1/2″, 3/4″, 11/12″, 3″ and 6″
- 8/64″ at 3″ and 6″

To find the exact position of defect in the testing material by ultrasonic pulse echo inspection, the time base of CRO be calibrated. For direct beam probe, the time base calibration can be achieved by placing the probe on calibration block to obtain multiple echoes from the 25 mm thickness. The time base can be adjusted using the echoes corresponding to 50, 100, 150, 200 and 250 mm blocks. Now the instrument time base is calibrated in terms of "metal distance" in steel. For other material, the metal distance is obtained by multiplying the steel metal distance with the ratio of velocity of sound in that material to the velocity of sound in steel. For angle beam probes, the time base can be obtained by using a similar procedure that is used for direct beam.

8.9 Applications of ultrasonics in NDT

The following are the applications of ultrasonics in non-destructive testing of materials:

(i) Ultrasonics are used in the testing of defects in welded, casted and forged materials used in industries.
(ii) Ultrasonic inspection is used in testing of any shaped and sized moulds. Water immersion technique is used to test irregular-shaped materials.
(iii) Ultrasonic testing is used in the semi-finished products such as bars, pipes, sheets, etc. in metal industries.
(iv) Ultrasonic pulse echo technique is used in accurate thickness measurements of materials.
(v) Ultrasonic testing is used in the routine inspection of air craft and rail vehicles for fatigue cracks.
(vi) Ultrasonic testing is carried in the periodic inspection of materials used in corrosion atmosphere and in chemical plants, also the products in operation.
(vii) Ultrasonic inspection has been carried in the finished products of metals and non-metals. The materials that cannot be tested with other NDT methods can also be tested using ultrasonic testing method.
(viii) In industries, mainly ultrasonics are used in the quality control of products by the assignment of surface and sub surface defects in metallic materials, such as locomotive axles, crankpins, booster axles, diesel driving axles and diesel traction motor shaft.

Multiple-choice Questions

1. The defects in welded, casted and forged materials can be detected without spoiling them by _______ of materials.

(a) non-destructive testing (b) destructive testing
(c) both a and b (d) none of the above

2. Non-destructive testing methods are _______.

(a) ultrasonics and radiography
(b) liquid penetration and thermal methods
(c) eddy current methods and magnetic particle testing
(d) all of the above

3. Ultrasonic methods of testing is used to find _______.

(a) cracks (b) voids (c) foreign material inclusions (d) all the above

4. Ultrasonic waves frequency range used in NDT is _______.

(a) 20 Hz to 2,000 Hz (b) 2,000 Hz to 20,000 Hz
(c) 200 KHz to 50 MHz (d) none of the above

5. The pulse generator in ultrasonic testing generates _______.

(a) high frequency alternating voltages of controlled amplitudes
(b) low frequency alternating voltages of controlled amplitudes
(c) high frequency alternating voltages of uncontrolled amplitudes
(d) none of the above

6. The echo receiver in ultrasonic testing receives _______ signals.

(a) sound (b) electrical (c) optical (d) magnetic

7. Using ultrasonic testing method _______.

(a) defects in metals can be estimated
(b) thickness of metal sheets and blocks can be estimated
(c) both a and b
(d) none of the above

8. The following is a piezoelectric crystal _______.

(a) quartz (b) potassium dihydrogen phosphate
(c) ammonium dihydrogen phosphate (d) all of the above

9. To get maximum energy out of the ultrasonic transducer _______ is placed between the crystal and the face of the transducer.

(a) an impedance matching (b) a coupling material
(c) capacitive matching (d) none of the above

10. The following material is an ultrasonic couplant _______.

(a) oil (b) water (c) glycerin (d) all the above

11. Ultrasonic inspection techniques are _______.

(a) pulse echo and transmission (b) resonance and frequency modulation
(c) both a and b (d) none of the above

12. Using ultrasonics _______ of the flaw in the material can be known.

(a) distance (b) size (c) shape (d) all the above

13. To detect surface detects _______ waves are used.

(a) surface or Rayleigh (b) transverse (c) shear (d) none of the above

14. Ultrasonic A-scan is a _______ scan of the material.

(a) one dimensional (b) two dimensional (c) three dimensional (d) none of the above

15. Ultrasonic B-scan provides _______.

 (a) depth of flaw from the surface of the material
 (b) length of flaw along the line of moment of probe
 (c) both a and b
 (d) none of the above

16. Ultrasonic C-scan produces _______ of the material.

 (a) plan view (b) elevation view (c) three-dimensional view (d) none of the views

17. The ultrasonic inspection standards are used for the _______ of measurement equipment.

 (a) precision (b) accuracy (c) both a and b (d) none of the above

18. The distance/area amplitude correction blocks set in the ultrasonic testing contains _______ blocks.

 (a) 5 (b) 10 (c) 12 (d) 20

Answers

1. a	2. d	3. d	4. c	5. a	6. b	7. c	8. d	9. a	10. d	11. c	12. d
13. a	14. a	15. c	16. a	17. c	18. b						

Review Questions

1. Draw the block diagram of ultrasonic flaw detector and explain the determination of flaw using it in a material.
2. Explain the principle of ultrasonic testing of materials.
3. What is an ultrasonic transducer? What are the different crystals used in it? Explain the use of an ultrasonic transducer.
4. What are ultrasonic contact transducer and immersion transducer? Explain the construction of an ultrasonic transducer.
5. Explain the necessity of couplant material in ultrasonic testing? How is this used in ultrasonic testing?
6. Explain the contact probe testing and immersion ultrasonic testing.
7. Describe the ultrasonic pulse echo testing technique used in testing of materials.
8. Describe the A-, B- and C-scans used in ultrasonic testing of materials.
9. Describe the inspection standards used in ultrasonic testing of materials.
10 What are the various applications of ultrasonics in non-destructive testing of materials?

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